1. B. \( \lim_{t \to 0} \frac{t^3}{\sin^2(2t)} = \lim_{t \to 0} \left( \frac{t}{\sin(2t)} \right)^3 \)

\[
= \frac{1}{8} \lim_{t \to 0} \left( \frac{2t}{\sin(2t)} \right)^3.
\]

Now, since \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \), it follows that \( \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1 \).

Thus \( \frac{1}{8} \lim_{t \to 0} \left( \frac{2t}{\sin(2t)} \right)^3 = \frac{1}{8}(1)^3 = \frac{1}{8} \).

2. C. Find \( f'(\pi/4) \) for \( f(x) = x \sin x \cos x \). By the product rule, \( f'(x) = \sin x \cos x + x(\cos^2 x - \sin^2 x) \).

Thus \( f'(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\pi}{4} - \frac{1}{2} = \frac{1}{2} \).

3. A. If \( g(x) = f(x^3) + (f(x))^3 \), \( f(1) = 2 \), \( f'(1) = -3 \), find \( g'(1) \). By the chain rule, \( g'(x) = f'(x^3)3x^2 + 3(f(x))^2 f'(x) \).

Thus \( g'(1) = f'(1)3(1)^2 + 3(1)^2 f'(1) = (-3)(3) + 3(1)(-3) = -45 \).

4. E. Find the slope of the parametric curve
\( x = e^{-5t}, \ y = t \cos t \) at the point \((1, 0)\). Note first that \( t = 0 \) yields the point \((1, 0)\). Thus the slope of the tangent line is
\( m = \frac{dy/dt}{dx/dt} \) evaluated at \( t = 0 \).

\[
m = \frac{dy/dt}{dx/dt} = \frac{\cos t - t \sin t}{-5e^{-5t}}.
\]

Substitute \( t = 0 \) yields
\( m = -1/5 \).

5. E. If \( g(x) \) is the inverse of \( f(x) = \sqrt{x^2 + x + 6} \), find \( g'(4) \).

We know that \( g'(4) = \frac{1}{f'(g(4))} \). Now, since \( f(2) = 4 \), it follows that \( g(4) = 2 \). Hence,
\[
g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(2)}.
\]

Now, \( f'(x) = \frac{1}{2}(x^2 + x + 6)^{-1/2}(3x) \).

Thus, \( f'(2) = \frac{1}{2}(16)^{-1/2}(13) = \frac{13}{8} \). Hence,
\[
g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(2)} = \frac{8}{13}.
\]

6. B. The graph of the curve \( y = x + \frac{1}{3} \cos 3x \) has a horizontal tangent at \( x = ? \). \( f(x) \) has a horizontal tangent where \( f'(x) = 0 \). Thus \( 1 - \sin(3x) = 0 \), hence \( \sin(3x) = 1 \) yielding \( x = \frac{\pi}{6} \).

7. D. An object is moving according to the equation \( s(t) = t^3 - 9t^2 + 15t + 8 \). When is the object moving in the negative direction? The object is moving in the negative direction when \( v(t) < 0 \). Now,
\( v(t) = s'(t) = 3t^2 - 18t + 15 = 3(t-5)(t-1) \).

Solve \( v(t) < 0 \) yields \( 1 < t < 5 \).

8. D. \( \lim_{x \to -\infty} \left( \frac{1}{3} \right) = \lim_{x \to -\infty} \left( \frac{1}{3} \right) = \frac{1}{3} = \infty \).

9. A. Find the slope of the tangent line to the curve \( y^3 - xy = 2x + 4 \) at the point \((1, 2)\). Differentiating implicitly, \( 3y^2 \frac{dy}{dx} - y - x \frac{dy}{dx} = 2 \). Solve for \( \frac{dy}{dx} \) yields
\( \frac{dy}{dx} = \frac{2+y}{3y^2-x} \). Substitute \( x = 1 \) and \( y = 2 \) gives us a slope of \( m = \frac{4}{11} \).

10. A. Find the linear approximation, \( L(x) \), of \( f(x) = \sqrt{x^2 + 9} \) at \( x = -4 \). The linear approximation for \( f(x) \) at \( x = -4 \) is
\( L(x) = f(-4) + f'(-4)(x - (-4)) \).

Thus, \( f(-4) = \sqrt{25} = 5 \). Since \( f'(x) = \frac{x}{\sqrt{x^2 + 9}} \), it follows that \( f'(-4) = -\frac{4}{5} \). Thus
\( L(x) = 5 - \frac{4}{5}(x + 4) \) or \( L(x) = \frac{9}{5} - \frac{4}{5}x \).

11. D. Find a unit tangent vector to the curve
\( r(t) = \left( \frac{2}{t} + 1, t\sqrt{t} + \frac{t}{2} \right) \) at \( t = 1 \).

Thus, the tangent vector at \( t = 1 \) is \( r'(1) = (-2, 2) \). Divide by the magnitude to make the tangent vector a unit vector yields
\( \left( \frac{-2}{\sqrt{8}}, \frac{2}{\sqrt{8}} \right) \).

12. Given that \( s(t) = e^{-3t} \cos(2t) \) is the equation of motion of an object, find
(i) The velocity at time \( t \). By the product and chain rule,
\( v(t) = s'(t) = -3e^{-3t} \cos(2t) + e^{-3t}(-2 \sin(2t)) = -e^{-3t}(3 \cos(2t) + 2 \sin(2t)) \).

(ii) The acceleration at time \( t \). \( a(t) = v'(t) = s''(t) = 3e^{-3t}(3 \cos(2t) + 2 \sin(2t)) - e^{-3t}(-6 \sin(2t) + 4 \cos(2t)) \).
13. Find the equation of the tangent line to the curve
\[ y = \sqrt{x + 8} + x \] at \( x = 1 \). The point of tangency is (1, 2). The slope of the tangent line is found by taking the derivative of \( y \) and then substituting \( x = 1 \).
\[
\frac{dy}{dx} = \frac{1}{2} \left( x + (8 + x)^{1/2} \right)^{-1/2} \left( 1 + \frac{1}{2} (8 + x)^{-1/2} \right).
\]
When \( x = 1 \), \( \frac{dy}{dx} = \frac{7}{24} \). Thus the equation of the tangent line is
\[ y - 2 = \frac{7}{24} (x - 1) \] or \[ y = \frac{7}{24} x + \frac{41}{24} \].

14. Use differentials (or a linear approximation) to estimate \((1.02)^{10}\). Let \( f(x) = x^{10}, a = 1 \) and therefore \( dx = 0.02 \). Using differentials,
\[
f(1.02) = (1.02)^{10} \approx f(1) + 10(0.02) = 1 + 10(0.02) = 1.2.
\]

15. The volume of a cube is increasing at a rate of 10 cubic centimeters per minute. How fast is the surface area increasing when the length of the edge is 30 cm?
The volume of a cube is \( V = x^3 \). The surface area is \( A = 6x^2 \). We want to express the surface area in terms of the volume. Solve \( V = x^3 \) for \( x \) yields \( x = \sqrt[3]{V} \).

Thus, \( A = 6V^{2/3} \), \( \frac{dA}{dt} = 4V^{-1/3} \frac{dV}{dt} \). Now, when \( x = 30 \), \( V = 30^3 \). Thus \( \frac{dA}{dt} = 4(30^3)^{-1/3}(10) = \frac{4}{3} \) square cm per minute.

16. Find \( f'(t) \) for the following functions:

   (i) \( f(t) = \tan^3(t^2 + a^2) \)
   \[ f'(t) = 3 \tan^2(t^2 + a^2) \sec^2(t^2 + a^2)(2t) \]

   (ii) \( f(t) = e^{t+\sqrt{t}} \)
   \[ f'(t) = (1 + \frac{1}{2\sqrt{t}})e^{t+\sqrt{t}} \]