Spring 2011 Math 151
Exam I Version B Solutions

1. C Set \( x \) and \( y \) components equal and solve for \( t \). \( t = 4 \) at \((2, -3)\), \( t = 0 \) at \((0, 1)\), \( t = 1 \) at \((1, 0)\), but \(-1 = \sqrt{7}\) has no solution, so the point not on the curve is \((-1, 0)\).

2. A \[ \frac{\cos x}{1 - \sin x} \cdot \frac{(1 + \sin x)}{(1 + \sin x)} = \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} \cdot \frac{1 + \sin x}{\cos x} = \frac{\cos x}{\sec x + \tan x}. \]

3. A Since \(-1 \leq \cos x \leq 1\), \(-\frac{1}{x} \leq \frac{\cos x}{x} \leq \frac{1}{x}\). Since \(\pm \frac{1}{x} \rightarrow 0 \) as \(x \rightarrow \infty\), by the Squeeze Theorem, the limit is 0.

4. B As we approach 4 from the left along the curve, the \(y\)-value approaches 2.

5. B Let \( f(x) = x^3 - x^2 + x \). \( f \) is continuous since it is a polynomial, and \( f(2) = 6, f(3) = 21 \), so \( f(2) < 10 < f(3) \). Therefore, by the Intermediate Value Theorem, there is a solution to \( f(x) = 10 \) on \([2, 3]\).

6. C \( a = (6i + 3j) - (3i - j) = 3i + 4j \). To form a unit vector \( \hat{a} \), multiply by the reciprocal of the magnitude: \( \hat{a} = \frac{1}{\sqrt{3^2 + 4^2}}(3i + 4j) = \frac{3}{5}i + \frac{4}{5}j \).

7. D \( \lim_{x \to 5^-} f(x) = 6 - 5 = 1 \). \( \lim_{x \to 5^+} f(x) = -8 + 2(5) = 2 \), and \( f(5) = 2 \). Therefore, since \( \lim_{x \to 5^-} f(x) = f(5) \neq \lim_{x \to 5^+} f(x) \), \( f \) is continuous only from the right.

8. B Factor \( x \) from numerator and denominator: \( \lim_{x \to -\infty} \frac{x(3 - \frac{8}{x})}{x(4 + \frac{9}{x})} = \frac{3}{4} \). Similarly, \( \lim_{x \to -\infty} \frac{x(3 - \frac{8}{x})}{x(4 + \frac{9}{x})} = \frac{3}{4} \).

9. E The angle between the gravity force (weight) and the motion of the block is 60\(^\circ\), so the work done is \( W = |F||D|\cos 60^\circ = (30)(20) \left( \frac{1}{2} \right) = 300 \text{ ft-lbs.} \)

10. B \( f'(3) = \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h} = \lim_{h \to 0} \frac{6h^2 - 4h + 7}{h} = 7 \). The equation of the line whose slope is 7 and which passes through the point \((3, 4)\) is \( y - 4 = 7(x - 3) \), or \( y = 7x - 17 \).

11. E \( \lim_{x \to 6} \frac{x(x-6)}{x-6} = \lim_{x \to 6} x = 6 \).

12. E (a) is both continuous and differentiable everywhere; (c) and (d) are not continuous at \( x = 0 \). From the graph of (c), we see that the function continuous but not differentiable at \( x = 0 \) is \( y = \sqrt{x} \).

13. \( f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{h \to 0} \frac{\sqrt{2} + 3x - \sqrt{5}}{x - 1} = \frac{\sqrt{2} + 3x + \sqrt{5}}{\sqrt{2} + 3x - \sqrt{5}} \)
\[ = \lim_{x \to 1} \frac{3x - 1}{(\sqrt{2} + 3x + \sqrt{5})} \]
\[ = \lim_{x \to 1} \frac{3(x - 1)}{(\sqrt{2} + 3x + \sqrt{5})} \]
\[ = \frac{3}{2\sqrt{5}}. \]

14. (a) \( \text{proj}_m n = \frac{m \cdot n}{|n|^2} n = \frac{4 + 2}{4^2 + 2^2}(4, 2) = \left\langle \frac{6}{5}, \frac{3}{5} \right\rangle \), so orth$_m n = (1, 1) - \left\langle \frac{6}{5}, \frac{3}{5} \right\rangle = \left\langle \frac{1}{5}, \frac{2}{5} \right\rangle.

(b) The point \((0, 0)\) is on the line, so let \( b = \langle 1, 1 \rangle - \langle 0, 0 \rangle = \langle 1, 1 \rangle \). The vector \( v = \langle 4, 2 \rangle \) is in the direction of the line, so \( v \perp = \langle -2, 4 \rangle \) is orthogonal to the line. The distance is found
by \(|\text{comp}_\perp b| = \frac{-2 + 4}{\sqrt{(-2)^2 + 4^2}} = \frac{2}{\sqrt{20}}\)

(NOTE that this is the magnitude of the vector in (a) above!)

15. Find a common denominator:

\[
\lim_{x \to 2} \frac{1}{x} - \frac{2}{x - 2} = \lim_{x \to 2} \frac{1}{x - 2} \cdot \frac{2 - x}{2x} = \lim_{x \to 2} -\frac{1}{2x} = -\frac{1}{4}.
\]

16. \(\mathbf{F}_1 = \langle 8 \cos 45^\circ, 8 \sin 45^\circ \rangle = \langle 4\sqrt{2}, 4\sqrt{2} \rangle\).
\(\mathbf{F}_2 = \langle 14 \cos 60^\circ, -14 \sin 60^\circ \rangle = \langle 7, -7\sqrt{3} \rangle\).

The resultant force is \(\mathbf{F}_1 + \mathbf{F}_2 = \langle 4\sqrt{2} + 7, 4\sqrt{2} - 7\sqrt{3} \rangle\). Therefore, the magnitude of the resultant force is

\[
\sqrt{(4\sqrt{2} + 7)^2 + (4\sqrt{2} - 7\sqrt{3})^2}.
\]

17. We use \(\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}\) to find the parametric equations. \(\mathbf{r}_0\) corresponds to the point at \(t = 1\): \(\mathbf{r}(1) = 5i + 5j\). \(\mathbf{v} = \mathbf{r}'(1) = \lim_{t \to 1} \frac{(5t)i + (8 - 3t^2)j - (5i + 5j)}{t - 1} = \lim_{t \to 1} \left( \frac{5t - 5}{t - 1} \right) i + \left( \frac{3 - 3t^2}{t - 1} \right) j = 5i - 6j\).

Therefore, the vector equation of the tangent line is \(\mathbf{r}(t) = (5i + 5j) + t(5i - 6j)\), so the parametric equations are \(\mathbf{r}(t) = (5 + 5t)i + (5 - 6t)j\), or \(x = 5 + 5t, y = 5 - 6t\).