GUIDELINES

1. In Part 1 (Problems 1–14), mark your responses on your ScanTron form using a No: 2 pencil. For your own record, mark your choices on the examination paper as well. ScanTrons will be collected at the conclusion of the examination; they will not be returned.

2. Calculators should not be used throughout the examination.

3. In Part 2 (Problems 15–19), present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to write your name, section number, and version letter of the examination on the ScanTron form.
Part 1 – Multiple Choice (56 points)

Each question is worth 4 points. Mark your responses on the ScanTron form and on the examination paper itself.

1. Let $P$, $Q$, and $R$ denote the vertices of a triangle. If $\overrightarrow{PQ} = c$ and $\overrightarrow{PR} = d$, what is $\overrightarrow{QR}$?
   
   (a) $-c - d$
   (b) $c - d$
   (c) $c + d$
   (d) $d$
   (e) $d - c$

2. Suppose that $a$, $b$, and $c$ are vectors. Which of the following expressions are meaningful? (i) $(a + b) \cdot c$, (ii) $a + (b \cdot c)$, (iii) $(a \cdot b) \cdot c$, (iv) $(a \cdot b)c$
   
   (a) only (ii) and (iii)
   (b) all four
   (c) only (ii)
   (d) only (iii)
   (e) only (i) and (iv)

3. Determine the value of the real number $x$ for which $\langle 3, 4 + 5x \rangle \cdot \langle x, -4 \rangle = 1$.
   
   (a) 0
   (b) 1
   (c) $-1$
   (d) $-15/23$
   (e) no such $x$ exists
4. Let \( \mathbf{a} = i - 2j, \mathbf{b} = -2i + 4j, \) and \( \mathbf{c} = 6i + 3j. \) Decide on the truth/falsity of each of the following statements: (i) \( \mathbf{a} \) is parallel to \( \mathbf{b}; \) (ii) \( \mathbf{a} \) is parallel to \( \mathbf{c}; \) (iii) \( \mathbf{a} \) is orthogonal to \( \mathbf{b}; \) (iv) \( \mathbf{b} \) is orthogonal to \( \mathbf{c}. \)

(a) (iv) is true, the rest are false
(b) (i) and (ii) are true, the rest are false
(c) (ii) and (iii) are true, the rest are false
(d) (i) and (iv) are true, the rest are false
(e) (i) is true, the rest are false

5. A constant force with the vector representation \( \mathbf{F} = 10i + 18j \) moves an object along a straight line from the point \((2, 3)\) to the point \((4, 9)\). Find the work done, if the distance is measured in meters and the magnitude of the force is measured in newtons.

(a) 74 J
(b) 96 J
(c) 202 J
(d) 128 J
(e) 162 J

6. Which of the following is a Cartesian equation of the parametric curve determined by the equations \( x(t) = \cos^2 t, \, y(t) = \sin t, \) for \( 0 \leq t \leq 2\pi? \)

(a) \( xy = 1 \)
(b) \( x^2 + y = 1 \)
(c) \( x + y^2 = 1 \)
(d) \( x + y = 1 \)
(e) \( x^2 + y^2 = 1 \)
7. Compute \( \lim_{x \to 2^-} \frac{x - 2}{|2x - 4|} \).

(a) does not exist

(b) 1/2

(c) -1

(d) 1

(e) -1/2

8. Compute \( \lim_{x \to 1} \left[ \frac{1}{x - 1} - \frac{2}{x^2 - 1} \right] \).

(a) 1

(b) +\(\infty\)

(c) -\(\infty\)

(d) 1/2

(e) 0

9. Calculate \( \lim_{t \to -\infty} \frac{\sqrt{2t^2 - t - 2}}{2t + 1} \).

(a) does not exist

(b) \(\sqrt{2}/2\)

(c) -1

(d) -\(\sqrt{2}/2\)

(e) 1
10. Let \( a \) be a real number. Suppose that \( \lim_{x \to a} f(x) = 5 \) and that \( \lim_{x \to a} [(x + a)f(x)] = 3 \). Determine the value of \( a \).

(a) 3/5
(b) 3/10
(c) 10/3
(d) 5/3
(e) 5

11. Consider the following curves:

(I) \( y = \frac{x^2 - x - 2}{x^2 - 1} \)  
(II) \( y = \frac{x^2 + x - 2}{x^2 - 1} \)  
(III) \( y = \frac{x}{\sin(\pi x)} \)

The line \( x = 1 \) is a vertical asymptote for

(a) (I) and (III) only
(b) (II) and (III) only
(c) (I) only
(d) (III) only
(e) all three

12. Which of the following curves has a horizontal asymptote?

(I) \( y = \cos x \)  
(II) \( y = \frac{2\sqrt{x}}{1 + \sqrt{x}} \)  
(III) \( y = \frac{2x}{1 + \sqrt{x}} \)

(a) none of the three
(b) (II) and (III) only
(c) (I) and (III) only
(d) (II) only
(e) all three
13. The function $f(x) = 1 - |x + 1|$ is
   (a) differentiable at every real number
   (b) not differentiable at 1
   (c) discontinuous at $-1$
   (d) not differentiable at $-1$
   (e) not differentiable at 0 and $-1$

14. The line $y = 3x - 6$ is the tangent to the graph of $f$ at the point $(2, 0)$. Compute
    \[ \lim_{x \to 2} \frac{x^2(2x + 1)f(x)}{x - 2}. \] Hint: Note that $f(x) = f(x) - f(2)$.
    (a) 0
    (b) 60
    (c) 24
    (d) 20
    (e) does not exist
Part 2 (49 points)
Present your solutions to the following problems (15–19) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. Consider the function

\[ f(x) = \sqrt{1 + 2x}, \quad \text{for } x \geq -\frac{1}{2}. \]

(i) (9 points) Use the definition of the derivative to compute \( f'(1) \). (Note: No credit will be given for using any other method, correct answer notwithstanding.)

(ii) (5 points) Obtain a Cartesian equation of the tangent to the curve \( y = f(x) \), when \( x = 1 \).
16. (7 points) Obtain a vector equation (that is, an equation in the standard form $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$, for $-\infty < t < \infty$) of the straight line which passes through the point $(3, -1)$, and is perpendicular to the vector $\langle 1, 7 \rangle$.

17. (7 points) Let $L_1$ denote the line given by the parametric equations $x(t) = -3 + 2t$, $y(t) = -1 + 4t$, for $-\infty < t < \infty$. Let $L_2$ denote the line given by the parametric equations $x(s) = -1 + s$, $y(s) = s$, for $-\infty < s < \infty$. Determine the point of intersection of $L_1$ and $L_2$. 
18. (7 points) Let $A(1, 0)$, $B(5, 1)$, and $C(2, 3)$ be the vertices of a triangle. Compute the cosine of the angle subtended at the vertex $B$ (i.e., the angle $\angle ABC$). The final answer need not be simplified.

19. Let $a$ be a real number, and let $g$ be the function defined as follows:

$$g(x) = \begin{cases} 
  x^2 + ax - 2, & \text{if } x < 2; \\
  0, & \text{if } x = 2; \\
  \frac{x^3 + x + 2a}{(2x-1)^2}, & \text{if } x > 2.
\end{cases}$$

(i) (4 points) Calculate $\lim_{x \to 2^-} g(x)$ and $\lim_{x \to 2^+} g(x)$. 

(ii) (6 points) Determine the value $a$ for which $\lim_{x \to 2} g(x)$ exists. Compute the limit.

(iii) (4 points) Is $g$ continuous at 2 for the value of $a$ determined in part (ii) above? Explain your reasoning clearly and concisely.
Rough(Scratch) Work

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