MATH 151, FALL SEMESTER 2012
COMMON EXAMINATION II - VERSION A

Last name, First name (print): ________________________________

Signature: ________________________________

Instructor’s name: ________________________________

Section No: ________________________________

GUIDELINES

- In Part 1 (Problems 1–14), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the examination paper as well.* ScanTrons will be collected at the conclusion of the examination; they will *not* be returned.

- In Part 2 (Problems 15–19), present your solutions in the space provided. *Show all your work* neatly and concisely, and *indicate your final answer clearly*. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

- Be sure to *write your name, section number, and version letter of the examination on the ScanTron form.*

- Calculators *should not be used* throughout the examination. Mobile phones *must be switched off*. Examinees found using a mobile phone during the course of the test are subject to being ejected from the examination hall, and to being given a zero on the examination.
Part 1 – Multiple Choice (56 points)

Each question is worth 4 points. Mark your responses on the ScanTron form and on the examination paper itself.

1. If \( f(x) = x^5 - x^3 + x - e^2 \), for \(-\infty < x < \infty\), compute \( f'(1) \).
   
   (a) 1 – 2e
   
   (b) 3 – e^2
   
   (c) 3 – 2e
   
   (d) 1
   
   (e) 3

2. Let \( f(x) = e^{x^2} \), for \(-\infty < x < \infty\). Compute \( f'(x) \).
   
   (a) \( x^2 e^{x^2-1} \)
   
   (b) \( e^{x^2} \)
   
   (c) \( e^{2x} \)
   
   (d) \( 2xe^{x^2} \)
   
   (e) \( 2xe^{x^2-1} \)

3. Let \( f(x) = \frac{1}{x + 1} \), for \( x \neq -1 \). Compute \( f^{(3)}(1) \).
   
   (a) \( \frac{3}{8} \)
   
   (b) \( -\frac{3}{8} \)
   
   (c) \(-6 \)
   
   (d) 6
   
   (e) \(-1/8 \)
4. Let \( f(x) = x^{4/3} \), for \(-\infty < x < \infty\). Find the \( x\)-coördinate of the point on the graph of \( f \) where the tangent is perpendicular to the line \( x + 4y = 1 \).

(a) 27
(b) 3
(c) -27
(d) -3
(e) \(-\frac{3^3}{16^3}\)

5. Let \( C \) be the curve determined by the vector-valued function \( \mathbf{r}(t) = (2\cos t, 3\sin t) \), for \( 0 \leq t \leq \pi \). Find the (co-ordinates of the) point on \( C \) where the tangent vector is parallel to \( \mathbf{i} = (1,0) \).

(a) (2, 0)
(b) (0, 3)
(c) (-2, 0)
(d) \((\sqrt{2}, 3\sqrt{2}/2)\)
(e) there is no such point

6. The vector-valued function \( \mathbf{r}(t) = (t \sin t, \cos(2t)) \), for \( 0 \leq t \leq 2\pi \), represents the position of a particle at time \( t \). Determine the acceleration vector at time \( t = \pi/2 \).

(a) (-1, -4)
(b) \(-\pi/2, -4\)
(c) (-1, 0)
(d) \((-\pi/2, 4)\)
(e) \((-1, 4)\)
7. Let \( f(x) = x^3 + 8(x + 1)^{3/2} \), for \( x \geq -1 \). Compute the quadratic approximation to \( f \), near \( x = 0 \).
   
   (a) \( 6x^2 + 12x + 8 \)
   
   (b) \( 3x^2 + 12x + 8 \)
   
   (c) \( 6x^2 + 12x \)
   
   (d) \( 3x^2 + 12x \)
   
   (e) \( 3x^2 + 8 \)

8. Suppose that \( F \) is a differentiable function, and let \( G \) be the function defined by \( G(x) = (x^2 + 1)F(x) \). If \( G'(1) = 2 \) and \( F'(1) = -3 \), what is the value of \( F(1) \)?
   
   (a) \( 4 \)
   
   (b) \( 8 \)
   
   (c) \( 0 \)
   
   (d) \( -4 \)
   
   (e) \( -8 \)

9. \( 2^{50} + 2^{50} = ? \)
   
   (a) \( 2^{100} \)
   
   (b) \( 2^{2500} \)
   
   (c) \( 4^{50} \)
   
   (d) \( 4^{100} \)
   
   (e) \( 2^{51} \)
10. Compute $\lim_{x \to -1^+} 2^{\frac{x}{x^2}}$.

(a) 2
(b) $\infty$
(c) $-\infty$
(d) $\sqrt{2}$
(e) 0

11. Calculate $\lim_{t \to 0} \left[ \frac{1}{t} - \frac{1}{t \cos t} \right]$.

(a) $-1$
(b) 0
(c) 1
(d) $-\infty$
(e) $+\infty$

12. Let $f(x) = \sec^2(x)$, for $0 \leq x < \pi/2$, and let $g$ denote the inverse of $f$. Determine the value of $g'(2)$.

(a) $2\sqrt{2}$
(b) 4
(c) $\sqrt{2}/4$
(d) 1/4
(e) 1/2
13. Let $f$ be a differentiable function, and let $G$ be the function defined by $G(x) = f(x)[f(f(x))]$. Compute the derivative of $G$ with respect to $x$.

(a) $f'(x)[f(f(x)) + f(x)f'(f(x))]$

(b) $f'(x)f'(f(x))$

(c) $[f'(x)]^2 f'(f(x))$

(d) $[f'(x)]^2 f'(f(x))$

(e) $3[f(x)]^2 f'(x)$

14. Suppose that the function $f$ is differentiable on $(0, \infty)$, and let $H(x) = f\left(\frac{1}{x}\right)$, for $x > 0$. Given that the line $y = 3x - 1$ is the tangent to the graph of $f$ at the point $(1, f(1))$, calculate the slope of the tangent to the graph of $H$ at the point $(1, H(1))$.

(a) 2

(b) 1/3

(c) 3

(d) 1/2

(e) -3
Part 2 (49 points)

Present your solutions to the following problems (15–19) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. Let $C$ be the curve described by the parametric equations

$$x(t) = t^3 - t + 8, \quad y(t) = \sqrt{t^2 + 5}, \quad \text{for} \quad 1 \leq t \leq 2.$$ 

(i) (8 points) Compute $x'(t)$ and $y'(t)$, for $1 \leq t \leq 2$.

$$x'(t) = 3t^2 - 1; \quad y'(t) = \frac{t}{\sqrt{t^2 + 5}}$$

(ii) (4 points) Compute the slope of the tangent to $C$ at the point corresponding to the parameter value $t$. Express the answer in terms of $t$.

$$\text{Slope} = \frac{y'(t)}{x'(t)} = \frac{t}{(3t^2 - 1) \sqrt{t^2 + 5}}.$$
16. (10 points) Let

\[
f(x) = \frac{x^3 \cos(2x)}{e^{3x} + e^{-x}}, \quad \text{for } -\infty < x < \infty.
\]

Compute \( f'(x) \). (It is not necessary to simplify the final answer.)

\[
f'(x) = \frac{\left[ e^{3x} + e^{-x} \right] \left[ 3x^2 \cos(2x) - 2x^3 \sin(2x) \right] - 2 \left[ 3 \cos(2x) \right] \left[ 3e^{3x} - e^{-x} \right]}{(e^{3x} + e^{-x})^2}
\]

17. (7 points) Suppose that the function \( f \) is differentiable throughout \([0, 2]\), and that \( f(x) \neq 0 \) for every \( 0 \leq x \leq 2 \). Given below is a table of values of \( f \) and its derivative:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>13</td>
</tr>
</tbody>
</table>

Let \( H(x) = \frac{1}{f(x)} \), for \( 0 \leq x \leq 2 \). Use the data above to compute the linear approximation to \( H \) near the point \( x = 1 \).

\[
H'(1) = \frac{-4}{81}, \quad \text{so} \quad L_{H}(1; x) = \frac{1}{9} - \frac{4}{81}(x-1)
\]
18. (10 points) A riverboat sails with constant velocity, along a straight path, parallel to a (straight) bank, at a fixed distance of 3 miles from the bank. The distance between the boat and the foot of a pole erected at the edge of the bank is increasing at the rate of 12 miles per hour, when the said distance is 5 miles. Determine the velocity of the boat. (Draw a figure; mark all the variables clearly.)

\[
\begin{align*}
\text{Given:} & \quad \frac{ds}{dt} = 12 \\
\text{Want:} & \quad \frac{dx}{dt} \\
\end{align*}
\]

\[s^2 = x^2 + 9 \Rightarrow 2s \frac{ds}{dt} = 2x \frac{dx}{dt}\]

\[\Rightarrow \frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}, \quad \text{when} \quad s = 5, \quad x = 4, \quad \text{so} \]

\[\frac{dx}{dt} = \frac{5}{4} \quad \Rightarrow \quad \frac{dx}{dt}
\]

19. (10 points) Given that

\[\sin(xy) + x^2 \sin(y^2) = 0,\]

employ implicit differentiation to compute \(\frac{dy}{dx}\).

\[
\cos(xy) [x y' + y] + x^2 [\cos(y^2) \cdot 2y y'] + 2x \sin(y^2) = 0
\]

\[\Rightarrow y' [x \cos(xy) + 2x^2 y \cos(y^2)] = -y \cos(xy) - 2x \sin(y^2)
\]

\[\Rightarrow y' = - \frac{y \cos(xy) + 2x \sin(y^2)}{x \cos(xy) + 2x^2 y \cos(y^2)}
\]

\[\]

9
LAST NAME (print): __________________________________________

FIRST NAME (print): _______________________________________

<table>
<thead>
<tr>
<th>QN</th>
<th>PTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

TOTAL