DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 16-22), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________

DO NOT WRITE BELOW!
PART I: Multiple Choice. 4 points each

1. Correct answer: (D) We know from vector addition, \( \overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR} \). Thus \( \overrightarrow{QR} = \overrightarrow{PR} - \overrightarrow{PQ} \), therefore \( \overrightarrow{QR} = \mathbf{d} - \mathbf{c} \).

2. Correct answer: (B) For the vectors \( \mathbf{a} \) and \( \mathbf{c} \):
   (i) \( (\mathbf{a} + \mathbf{b}) \cdot \mathbf{c} \) is the dot product of two vectors, hence meaningful.
   (ii) \( \mathbf{a} + (\mathbf{b} \cdot \mathbf{c}) \) is a vector plus a scalar, so it is meaningless.
   (iii) \( (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c} \) is the dot product of a scalar and a vector, hence is meaningless.
   (iv) \( (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \) is a vector multiplied by a scalar, hence is meaningful. Thus only (i) and (iv) are true.

3. Correct answer: (D) \( \langle 3, 4 + 5x \rangle \cdot \langle x, -4 \rangle = 1 \implies 3x + (4 + 5x)(-4) = 1 \). This yields \( x = -1 \).

4. Correct answer: (A) For \( \mathbf{a} = (1, -2) \), \( \mathbf{b} = (-2, 4) \), and \( \mathbf{c} = (6, 3) \):
   (i) \( \mathbf{b} = -2\mathbf{a} \), thus \( \mathbf{a} \) is parallel to \( \mathbf{b} \). (i) is therefore TRUE.
   (ii) \( \mathbf{a} \cdot \mathbf{c} = 0 \), thus \( \mathbf{a} \) is perpendicular to \( \mathbf{c} \). (ii) is therefore FALSE.
   (iii) Since \( \mathbf{a} \) is parallel to \( \mathbf{b} \), (iii) is FALSE.
   (iv) \( \mathbf{b} \cdot \mathbf{c} = 0 \), thus \( \mathbf{b} \) is perpendicular to \( \mathbf{c} \). (iv) is therefore TRUE.

5. Correct answer: (C) The work done in moving an object from the point \( P(2, 3) \) to the point \( Q(4, 9) \) via force \( \langle 10, 18 \rangle \) is defined as \( W = \mathbf{F} \cdot \mathbf{D} \), where \( \mathbf{D} = \overrightarrow{PQ} = (2, 6) \). Therefore \( W = (10, 18) \cdot (2, 6) = 20 + 108 = 128 \) Joules.

6. Correct answer: (E). For \( x(t) = \cos^2 t \) and \( y(t) = \sin t \), since \( \cos^2 t + \sin^2 t = 1 \), \( x(t) + (y(t))^2 = 1 \), thus \( x + y^2 = 1 \).

7. Correct answer: (A) First note for \( x < 2 \), \( |x - 2| = -(x - 2) \).
   Now, \( \lim_{x \to 2^-} \frac{x - 2}{2|x - 2|} = \lim_{x \to 2^-} \frac{x - 2}{2(x - 2)} = \frac{1}{2} \).

8. Correct answer: (E) \( \lim_{x \to 1} \left( \frac{1}{x - 1} - \frac{2}{x^2 - 1} \right) = \lim_{x \to 1} \frac{x + 1 - 2}{x^2 - 1} = \lim_{x \to 1} \frac{x - 1}{(x + 1)(x - 1)} = \frac{1}{2} \).

9. Correct Answer: (E) \( \lim_{t \to \infty} \frac{\sqrt{2t^2 - t} - 2}{2t + 1} = \lim_{t \to -\infty} \frac{\sqrt{2t^2 - t} + 2}{2t + 1} \).
   Now, we know \( \sqrt{2} = |t| = -t \) since \( t < 0 \).
   \[
   \lim_{t \to -\infty} \frac{\sqrt{2t^2 - t} - 2}{2t + 1} = \lim_{t \to -\infty} \frac{\sqrt{2t^2 - t} + 2}{2t + 1} = \lim_{t \to -\infty} \frac{-\sqrt{2} \cdot \sqrt{t^2} + 2}{2t + 1} = \lim_{t \to -\infty} \frac{-\sqrt{2} \cdot t - 2}{2t + 1} = \lim_{t \to -\infty} \frac{-\sqrt{2} \cdot t + 2}{2t + 1} = \lim_{t \to -\infty} \frac{-\sqrt{2} \cdot t - 2}{2t + 1} = -\frac{\sqrt{2}}{2}.
   \]

10. Correct Answer: (B) \( \lim_{x \to a} (x + a) f(x) = 2a (\lim_{x \to a} f(x)) = 2a(5) \). Now, since \( \lim_{x \to a} (x + a) f(x) \) was given to be 3, we have \( 10a = 3 \), thus \( a = \frac{3}{10} \).

11. Correct answer: (E) \( \frac{x^2 - x - 2}{x^2 - 1} = \frac{(x - 2)(x + 1)}{(x + 1)(x - 1)} \).
   This function has a vertical asymptote at \( x = 1 \) and a hole in the graph at \( x = -1 \).
   \( \frac{x^2 + x - 2}{x^2 - 1} = \frac{(x + 2)(x - 1)}{(x + 1)(x - 1)} \).
   This function has a vertical asymptote at \( x = -1 \) and a hole in the graph at \( x = 1 \).
   \( \frac{x}{\sin(\pi x)} \) has a vertical asymptote at \( x = 1 \).
12. Correct answer: (D)

(I) \( \lim_{x \to \pm \infty} \cos(x) \) does not exist, thus this function has NO horizontal asymptotes.

(II) \( \lim_{x \to \infty} \frac{2\sqrt{x}}{1 + \sqrt{x}} = 2 \), thus \( y = 2 \) is a horizontal asymptote.

(III) \( \lim_{x \to \infty} \frac{2x}{1 + \sqrt{x}} = \infty \), thus this function has NO horizontal asymptote.

13. Correct answer: (C) By viewing the graph of \( f(x) \), we can see that \( f(x) \) is continuous everywhere but not differentiable at \( x = -1 \).

14. Correct answer: (C) Since \( y = 3x - 6 \) is the tangent line to the graph of \( f(x) \) at \( x = 2 \), we know that \( f'(2) = 3 \).

Hence \( \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = 3 \). Also, we are given that \( f(2) = 0 \).

Thus \( \lim_{x \to 2} \frac{x^2(2x + 1)f(x)}{x - 2} = \lim_{x \to 2} x^2(2x + 1) \frac{f(x) - 0}{x - 2} = \lim_{x \to 2} x^2(2x + 1) \frac{f(x) - f(2)}{x - 2} \cdot 4(5)(3) = 60 \).
PART II: Work Out

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. (i) \( f(x) = \sqrt{1 + 2x} \). \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{1 + 2(x+h) - \sqrt{1 + 2x}}}{h} \)

\[
= \lim_{h \to 0} \frac{1 + 2(x + h) - (1 + 2x)}{h(\sqrt{1 + 2(x+h) + \sqrt{1 + 2x})}} = \frac{2}{\sqrt{1 + 2} + \sqrt{1 + 2}} = \frac{1}{\sqrt{1 + 2}}
\]

(ii) \( m = f'(1) = \frac{1}{\sqrt{3}} \). The equation of the tangent line at \( x = 1 \) is \( y - f(1) = f'(1)(x - 1) \). Since \( f(1) = \sqrt{3} \), the tangent line is \( y - \sqrt{3} = \frac{1}{\sqrt{3}}(x - 1) \)

16. The line in question is perpendicular to the vector \( \langle 1, 7 \rangle \), thus \( \mathbf{v} = \langle -7, 1 \rangle \). Thus the equation of the line is \( r_0 + t\mathbf{v} = \langle 3, -1 \rangle + t\langle -7, 1 \rangle = \langle 3 - 7t, -1 + t \rangle \)

17. To find the point of intersection, we will first equate components. EQ1: \(-3 + 2t = -1 + s \) and EQ2: 
\[ -1 + 4t = s. \]
Replace \( s \) with \(-1 + 4t \) in EQ1. This yields \(-3 + 2t = -1 + 1 + 4t \). Solve for \( t \) gives \( t = -\frac{1}{2} \), and hence \( s = -3 \).

Secondly, to find the point of intersection, we can replace \( t \) with \(-\frac{1}{2} \) in the first line OR \( s = -3 \) into the second line. Both of these yield the point \((-4, -3)\), and this is the intersection point.

18. \( A(1, 0) \), \( B(5, 1) \) and \( C(2, 3) \) are vertices of \( \triangle ABC \). To find the cosine of the angle at the vertex \( B \), we will solve the equation \( \cos \beta = \frac{\mathbf{BA} \cdot \mathbf{BC}}{|\mathbf{BA}| |\mathbf{BC}|} \) where \( \mathbf{BA} = \langle -4, -1 \rangle \) and \( \mathbf{BC} = \langle -3, 2 \rangle \). Thus \( \cos \beta = \frac{-4 \cdot -1 \cdot -3}{\sqrt{17}\sqrt{13}} = \frac{10}{\sqrt{17}\sqrt{13}} \)

19. (10 pts) Consider \( g(x) = \begin{cases} 
  x^2 + ax - 2 & \text{if } x < 2 \\
  0 & \text{if } x = 2 \\
  \frac{x^3 + x + 2a}{(2x - 1)^3} & \text{if } x > 2
\end{cases} \)

(i) \( \lim_{x \to 2^-} g(x) = 2 + 2a \) and \( \lim_{x \to 2^+} g(x) = \frac{10 + 2a}{9} \)

(ii) \( g(x) \) exists provided \( \lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x) \). Thus \( 2 + 2a = \frac{10 + 2a}{9} \). Solving for \( a \), we find \( a = -\frac{1}{2} \). For this value of \( a \), \( \lim_{x \to 2^+} g(x) = 1 \). However, \( g(2) = 0 \), so this value of \( a \) does not make \( g(x) \) continuous.