MATH 151, FALL 2013
COMMON EXAM I - VERSION A

LAST NAME: ___________________________ FIRST NAME: ___________________________
INSTRUCTOR: __________________________
SECTION NUMBER: _____________
UIN: ________________________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 16-22), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 3 points each

1. If \( \mathbf{a} = (1, 1) \), \( \mathbf{b} = (2, 1) \) and \( \mathbf{c} = (4, -3) \), what value of \( t \) satisfies \( \mathbf{c} = s\mathbf{a} + t\mathbf{b} \), where \( s \) and \( t \) are scalars?

(a) \( t = -7 \)
(b) \( t = 7 \)
(c) \( t = -10 \)
(d) \( t = 1 \)
(e) \( t = 2 \)

\[
\begin{align*}
\mathbf{c} &= s\mathbf{a} + t\mathbf{b} \\
\begin{bmatrix} 4 \\ -3 \end{bmatrix} &= \begin{bmatrix} s+2t \\ s+t \end{bmatrix} \\
\begin{cases}
5+2t &= 4 \\
5+t &= -3
\end{cases}
\Rightarrow \begin{cases}
s+2t &= 4 \\
5+t &= -3
\end{cases} \\
s &= \frac{4-2t}{1} \\
5+t &= -3 \\
t &= -7
\end{align*}
\]

2. Find \( \lim_{x \to 3} \frac{|x-3|}{x^2-2x-3} \) for \( x < 3 \), \( |x-3| = -(x-3) \)

(a) \( \frac{1}{4} \)
(b) 0
(c) \( \infty \)
(d) \( \frac{1}{4} \)
(e) The limit does not exist

\[
\begin{align*}
\lim_{x \to 3^-} \frac{|x-3|}{x^2-2x-3} &= \lim_{x \to 3^-} \frac{-(x-3)}{x^2-2x-3} \\
&= \lim_{x \to 3^-} \frac{x-3}{(x-3)(x+1)}
\end{align*}
\]

3. Find the vector projection of \((-3, 1)\) onto \((2, 5)\).

(a) \( \left\langle \frac{-2}{\sqrt{29}}, \frac{-5}{\sqrt{29}} \right\rangle \)
(b) \( \left\langle \frac{3}{10}, \frac{1}{10} \right\rangle \)
(c) \( \left\langle \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}} \right\rangle \)
(d) \( \left\langle \frac{2}{29}, \frac{-5}{29} \right\rangle \)
(e) \( \left\langle \frac{11}{29}, \frac{55}{29} \right\rangle \)

\[
\begin{align*}
\text{proj}_\mathbf{a} \mathbf{b} &= \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \\
&= \left\langle \frac{2}{29}, \frac{-5}{29} \right\rangle \\
&= \frac{2 \cdot 2 - 5 \cdot 1}{29} \\
&= \left\langle \frac{4}{29}, \frac{-25}{29} \right\rangle
\end{align*}
\]

4. Find \( \lim_{x \to -1^-} \frac{x-2}{x+1} \)

(a) \( \infty \)
(b) 0
(c) 1
(d) \( -\infty \)
(e) The limit does not exist

\[
\begin{align*}
\lim_{x \to -1^-} \frac{x-2}{x+1} &= \infty \\
\text{since } \frac{x-2}{x+1} &> 0 \text{ for } x \to -1^-
\end{align*}
\]

2
5. Given the points $A(0,1)$, $B(2,0)$ and $C(3,-4)$, find the angle, $\alpha$, located at the vertex $A$. That is, $\angle BAC$.

(a) $\gamma = \arccos \left( \frac{11}{\sqrt{170}} \right)$

(b) $\alpha = \arccos \left( \frac{-11}{\sqrt{170}} \right)$

(c) $\alpha = \arccos \left( \frac{1}{\sqrt{170}} \right)$

(d) $\gamma = \arccos \left( \frac{-6}{\sqrt{85}} \right)$

(e) $\gamma = \arccos \left( \frac{2}{\sqrt{85}} \right)$

\[ \cos \alpha = \frac{\langle 2,-1 \rangle \cdot \langle 3,-5 \rangle}{1 \cdot |\langle 2,-1 \rangle| \cdot |\langle 3,-5 \rangle|} = \frac{11}{\sqrt{5} \cdot \sqrt{34}} \]

\[ \alpha = \arccos \left( \frac{11}{\sqrt{170}} \right) \]

6. Find $\lim_{t \to 4} r(t)$ where $r(t) = \left\langle 2t + 1, \frac{\sqrt{t+5} - 3}{t-4} \right\rangle$.

(a) $(9,0)$

(b) $(9,1)$

(c) $\left\langle \frac{9+1}{6} \right\rangle$

(d) $\left\langle 9, -\frac{1}{6} \right\rangle$

(e) $\left\langle 9, -\frac{1}{2} \right\rangle$

\[ \lim_{t \to 4} (2t + 1) = 9 \]

\[ \lim_{t \to 4} \frac{\sqrt{t+5} - 3}{t-4} \cdot \frac{\sqrt{t+5} + 3}{\sqrt{t+5} + 3} = \lim_{t \to 4} \frac{t+5 - 9}{(t-4)(\sqrt{t+5} + 3)} = \lim_{t \to 4} \frac{t-4}{(\sqrt{t+5} + 3)} = 6 \]

7. Find the horizontal and vertical asymptotes for $f(x) = \frac{(2-x)(3x+1)}{x^2 - 4}$.

(a) $x = -3$, $y = -2$

(b) $y = -3$, $x = 2$, $x = -2$

(c) $x = -3$, $y = 2$, $y = -2$

(d) $y = -3$, $x = -2$

(e) $y = 3$, $x = -2$

Horizontal asymptote: $y = -3$

Vertical asymptote: $x = -2$

8. Find a unit vector in the direction of $a - b$ where $a = 1 - 3j$ and $b = -5j$.

(a) $\frac{6}{\sqrt{45}}i - \frac{3}{\sqrt{45}}j$

(b) $\frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j$

(c) $\frac{1}{\sqrt{65}}i - \frac{2}{\sqrt{65}}j$

(d) $\frac{2}{\sqrt{5}}i - \frac{1}{\sqrt{5}}j$

(e) $\frac{8}{\sqrt{65}}i - \frac{1}{\sqrt{65}}j$

\[ a = \langle 1, -3 \rangle \]

\[ b = \langle 0, -5 \rangle \]

\[ a - b = \langle 1, 2 \rangle \]

\[ \vec{u} = \frac{\frac{1}{2} + \frac{2}{3}j}{\sqrt{5}} \]

\[ \vec{u} = \frac{i + 2j}{1 + 2j} \]

\[ \vec{u} = \frac{1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j \]
9. Which interval contains a solution to the equation $x^3 + x = 3$?

(a) $[-1, 0]$  
(b) $[0, 2]$  
(c) $[0, 1]$  
(d) $[-2, -1]$  
(e) $[2, 4]$  


\[ f(x) = x^3 + x, \text{ a continuous function} \]

\[ N = 3 \]

since $f(0) = 0 < 3$ and $f(2) = 10 > 3$, a solution to $f(x) = 3$ exists on $[0, 2]$.  

10. Consider $f(x) = \begin{cases} x^2 + 5x + 1 & \text{if } x < -1 \\ 3 & \text{if } x = -1 \\ 2x - 1 & \text{if } x > -1 \end{cases}$ : Why is $f(x)$ not continuous at $x = -1$?

(a) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1} f(x) \neq f(-1)$.
(b) $f(x)$ is not continuous at $x = -1$ because $f(-1)$ does not exist.
(c) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1} f(x)$ does not exist.
(d) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1^+} f(x)$ does not exist.
(e) $f(x)$ is not continuous at $x = -1$ because $\lim_{x \to -1^-} f(x)$ does not exist.

\[ \lim_{x \to -1} f(x) = 3, \text{ but } f(-1) = 3 \]

11. A horizontal force of 20 pounds is acting on a box as it is pushed up a ramp that is 5 feet long and inclined at an angle of $60^\circ$ above the horizontal. Find the work done on the box.

(a) $50\sqrt{3}$ foot pounds  
(b) $50\sqrt{2}$ foot pounds  
(c) 100 foot pounds  
(d) 10 foot pounds  
(e) 50 foot pounds

\[ W = 1510 \cos 60^\circ = (30 \text{ lbs})(5 \text{ feet}) \left( \frac{1}{2} \right) = 75 \text{ ft-lbs} \]

12. Find $\lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 4)(x^2 - 4)}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 4)(x + 2)(x - 2)}{x - 2}$

(a) 0  
(b) $\infty$  
(c) 4  
(d) 32  
(e) 1

\[ = 32 \]

4
13. Find the value of $z$ so that the vectors $(4, x + 1)$ and $(x, 3)$ are perpendicular.

(a) $z = 0$
(b) $z = -\frac{7}{3}$
(c) $z = -\frac{3}{7}$
(d) $z = -\frac{1}{7}$
(e) $z = \frac{1}{7}$

\[ \langle 4, x+1 \rangle \cdot \langle x, 3 \rangle = 0 \]
\[ 4x + 3x + 3 = 0 \]
\[ 7x = -3 \]
\[ x = -\frac{3}{7} \]

14. Find the average rate of change of $f(t) = \sqrt{2t} + 3$ from $t = 1$ to $t = 3$.

(a) $3 - \sqrt{5}$
(b) $\frac{3 + \sqrt{5}}{2}$
(c) $\frac{\sqrt{5} - 3}{2}$
(d) $3 + \sqrt{5}$
(e) $\frac{3 - \sqrt{5}}{2}$

\[ \text{AROC} = \frac{f(3) - f(1)}{3 - 1} = \frac{3 - \sqrt{5}}{2} \]

15. Consider the graph of $f(x)$ given:

Which of the following is the graph of its derivative, $f'(x)$?

(a) 
(b) 
(c) 
(d) 
(e)
PART II: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Consider the line \( x = 8 - 2t, y = 14 + 7t. \)

(i) (2 pts) Find a vector parallel to the line.

\[ \mathbf{u} = \langle -2, 7 \rangle \text{ or any scalar multiple of this vector} \]

(ii) (2 pts) Find a vector perpendicular to the line.

\[ \mathbf{u}^\perp = \langle -7, -2 \rangle \text{ or any scalar multiple of this vector} \]

(iii) (2 pts) Find the \( x \) and \( y \) intercepts of the line.

\[ x\text{-intercept: } y = 0 \rightarrow 0 = 14 + 7t \]
\[ t = -2 \rightarrow x = 12 \]

\[ y\text{-intercept: } x = 0 \rightarrow 0 = 8 - 2t \rightarrow t = 4 \rightarrow y = 42 \]

17. (8 pts) Find \( \lim_{x \to \infty} (\sqrt{x^2 + 6x - 1} - x). \)

\[ \lim_{x \to \infty} \frac{(\sqrt{x^2 + 6x - 1} - x)}{+x} \]

\[ = \lim_{x \to \infty} \frac{x^2 + 6x - 1 - x^2}{\sqrt{x^2 + 6x - 1} + x} \]

\[ = \lim_{x \to \infty} \frac{6x - 1}{\sqrt{x^2 + 6x - 1} + x} \]

\[ \lim_{x \to \infty} \frac{6 - \frac{1}{x}}{\sqrt{1 + \frac{6}{x} - 1} + 1} \]

\[ = \lim_{x \to \infty} \frac{6}{\sqrt{6 + 1}} \]

\[ = \frac{6}{2} \]

\[ = 3 \]
18. (5 pts) If \( f(2) = 3 \) and \( f'(2) = -7 \), find the equation of the tangent line to the graph of \( f(x) \) at \( x = 2 \).

\[
m = f'(2) = -7
\]

Point \( (a, f(a)) = (2, 3) \)

\[
y - 3 = -7(x - 2) \quad \Rightarrow \quad y = -7x + 17
\]

19. (10 pts) For \( f(x) = \frac{1}{2x + 1} \), find \( f'(x) \) using the limit definition of the derivative.

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \frac{1}{2x + 2h + 1} - \frac{1}{2x + 1}
\]

\[
= \lim_{h \to 0} \frac{2x + 1 - (2x + 2h + 1)}{h(2x + 2h + 1)(2x + 1)}
\]

\[
= \lim_{h \to 0} \frac{-2h}{h(2x + 2h + 1)(2x - 1)}
\]

\[
= \frac{-2}{(2x + 1)^2}
\]
20. (10 pts) Consider \( f(x) = \begin{cases} \frac{1}{2} & \text{if } x = 5 \\ cx + 2 & \text{if } x > 5 \\ cx^2 - 4 & \text{if } x < 5 \end{cases} \)

(i) Find \( \lim_{x \to 5^+} f(x) \) in terms of \( c \).

\[
\lim_{x \to 5^+} f(x) = 5c + 2
\]

(ii) Find \( \lim_{x \to 5^-} f(x) \) in terms of \( c \).

\[
\lim_{x \to 5^-} f(x) = 25c - 4
\]

(iii) For what value of \( c \) does \( \lim_{x \to 5} f(x) \) exist?

\[
5c + 2 = 25c - 4
\]

\[
6 = 20c
\]

\[
c = \frac{6}{20} = \frac{3}{10}
\]

(iv) For the value of \( c \) found above, what is \( \lim_{x \to 5} f(x) \)?

\[
\lim_{x \to 5} f(x) = \frac{15}{10} + 2 = \frac{7}{2}
\]

(v) For the value of \( c \) above, is \( f(x) \) continuous at \( x = 5 \)? Support your answer.

\[
\lim_{x \to 5} f(x) = \frac{7}{2}, \quad f(5) = \frac{1}{2}
\]

Since \( \lim_{x \to 5} f(x) \neq f(5) \), \( f(x) \) is not continuous at \( x = 5 \).
21. (8 pts) Two forces $F_1$ and $F_2$ with magnitudes 6 lbs and 4 lbs, respectively, act on an object at a point $P$ as shown.

(i) Find the vector $F_1$. Evaluate trig functions.

$$F_1 = \left< -6 \frac{\sqrt{2}}{2}, 6 \frac{\sqrt{2}}{2} \right>$$

(ii) Find the vector $F_2$. Evaluate trig functions.

$$F_2 = \left< 4 \frac{\sqrt{3}}{2}, 4 \frac{1}{2} \right>$$

(iii) Find the resultant force, $F$, acting on the object.

$$F = F_1 + F_2$$

$$F = \left< -3\sqrt{3} + 2\sqrt{3}, 3\sqrt{3} + 2 \right>$$
22. Consider the curve $x = 3 + \cos t$, $y = -1 + \sin t$.
   (i) (4 pts) Eliminate the parameter to find a Cartesian equation.

   $$x - 3 = \cos t$$
   $$y + 1 = \sin t$$

   $$(x - 3)^2 + (y + 1)^2 = 1.$$

   (ii) (4 pts) Sketch the curve on the grid below.