MATH 151, SPRING 2014
COMMON EXAM I - VERSION B

LAST NAME(print): ___________________________ FIRST NAME(print): _______________________________

INSTRUCTOR: ________________________________

SECTION NUMBER: ____________________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-17), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 18-23), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 3 points each

1. Find the average rate of change of \( f(x) = x^2 + 6 \) from \( x = -3 \) to \( x = 1 \).

   (a) 8
   (b) 2
   (c) -4
   (d) -2
   (e) None of these

   \[
   \frac{f(1) - f(-3)}{1 - (-3)} = \frac{7 - 15}{1 + 3} = \frac{-8}{4} = -2
   \]

2. If \( \theta \) is the angle between the vectors \( \mathbf{m} = (1, 2) \) and \( \mathbf{n} = (-3, 4) \), then \( \cos(\theta) = \)

   (a) \( \frac{1}{4} \)
   (b) None of these.
   (c) \( \frac{2}{\sqrt{3}} \)
   (d) \( \frac{1}{\sqrt{2}} \)
   (e) \( \frac{1}{\sqrt{5}} \)

   \[
   \mathbf{m} \cdot \mathbf{n} = |\mathbf{m}| |\mathbf{n}| \cos \theta
   \]
   \[
   1(-3) + 2(4) = \sqrt{5} \cdot \sqrt{5} \cos \theta
   \]
   \[
   5 = 5 \cos \theta
   \]
   \[
   \cos \theta = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}
   \]

3. If \( \sin(x) = \frac{4}{\sqrt{65}} \) and \( x \) is in Quadrant II find the value of \( \tan(x) \).

   (a) \( \frac{4}{7} \)
   (b) \( \frac{7}{4} \)
   (c) \( \frac{4}{7} \)
   (d) None of these.
   (e) \( \frac{7}{4} \)

   \[
   \tan(x) = \frac{-4}{7} = -\frac{4}{7}
   \]

   \[
   -\sqrt{(\cos^2 x - y^2)} = -\sqrt{65 - 16} = -7
   \]

4. Which of the following vectors is parallel to the line \( y = -\frac{3}{7} x - 10 \)

   (a) \( 7\mathbf{i} + 3\mathbf{j} \)
   (b) \(-3\mathbf{i} + 7\mathbf{j} \)
   (c) \(-14\mathbf{i} + 6\mathbf{j} \)
   (d) \( 3\mathbf{i} + 7\mathbf{j} \)
   (e) \( \mathbf{i} - 10\mathbf{j} \)

   \[
   \mathbf{m} = -\frac{3}{7}
   \]

   Slope vector = \( \langle 7, -3 \rangle \)

   -14\mathbf{i} + 6\mathbf{j} is parallel since

   \[
   -2 \langle 7, -3 \rangle = \langle -14, 6 \rangle
   \]
5. If \( \mathbf{a} = (1, -4) \) and \( \mathbf{b} = (2, 3) \), find \( 3\mathbf{a} - 2\mathbf{b} \).

(a) None of these
(b) \((-1, -18)\)
(c) \((7, -6)\)
(d) \((5, 6)\)
(e) \((3, -1)\)

\[
3\mathbf{a} - 2\mathbf{b} = 3\langle 1, -4 \rangle - 2\langle 2, 3 \rangle = \langle 3, -12 \rangle - \langle 4, 6 \rangle = \langle -1, -18 \rangle.
\]

6. Given the points \( P(1, 14) \) and \( Q(6, 2) \), find a vector that is twice the length and in the same direction as \( \overrightarrow{PQ} \).

(a) \(-16\mathbf{i} + 24\mathbf{j}\)
(b) \(-5\mathbf{i} + 12\mathbf{j}\)
(c) \(-\frac{10}{13}\mathbf{i} + \frac{24}{13}\mathbf{j}\)
(d) \(10\mathbf{i} - 24\mathbf{j}\)
(e) \(\frac{10}{13}\mathbf{i} - \frac{24}{13}\mathbf{j}\)

\[
\overrightarrow{PQ} = \langle -5, 12 \rangle,
\]

\[
2\overrightarrow{PQ} = \langle -10, 24 \rangle.
\]

7. The vector \( \mathbf{a} \) is shown in the figure to the right. Which of the following represents \( \mathbf{a} \)?

(a) \((-40 \sin(20), 40 \cos(20))\)
(b) \((-40 \cos(20), 40 \sin(20))\)
(c) \((40 \cos(70), 40 \sin(70))\)
(d) \((40 \sin(20), 40 \cos(20))\)
(e) \((- \cos(70), \sin(70))\)

\[
|\mathbf{a}| = 40
\]

8. A line is given by the parametric equations \( x = 2 + 3t, y = 4 + 12t \). Find the slope of this line.

(a) \(-4\)
(b) \(\frac{1}{4}\)
(c) \(\frac{1}{2}\)
(d) \(\frac{3}{4}\)
(e) \(2\)

\[
r(t) = \langle 2 + 3t, 4 + 12t \rangle = \langle 2, 4 \rangle + t \langle 3, 12 \rangle
\]

\[
r = r_0 + tv
\]

\[
v = \langle 3, 12 \rangle \implies m = \frac{\frac{12}{3}}{3} = 4
\]
9. Here is the graph of a function \( f(x) \). Which of the following is false?

(a) \( f(x) \) is continuous from the right at \( x = 1 \).
(b) \( f(1) = 4 \).
(c) \( \lim_{x \to 2} f(x) \) does not exist.
(d) \( \lim_{x \to 1} f(x) = 2 \)
(e) \( f(x) \) has a removable discontinuity at \( x = 2 \).

\[
\lim_{x \to 2} f(x) = 3
\]

10. Find the vector projection of \( \langle 3, 4 \rangle \) onto \( \langle -1, 5 \rangle \).

(a) \( \begin{pmatrix} 51 \\ 68 \\ 25 \end{pmatrix} \)
(b) \( \begin{pmatrix} -17 \\ 85 \\ \sqrt{26} \end{pmatrix} \)
(c) \( \begin{pmatrix} 51 \\ 68 \\ \sqrt{26} \end{pmatrix} \)
(d) \( \begin{pmatrix} 51 \\ 68 \\ 5 \end{pmatrix} \)
(e) \( \begin{pmatrix} -17 \\ 85 \\ 26 \end{pmatrix} \)

\[ |a| = \sqrt{(-1)^2 + 5^2} \]
\[ = \sqrt{26} \]

\[
\text{Proj}_b a = \frac{a \cdot b}{|a|^2} a
\]
\[
= \frac{3(-1) + 4(5)}{(\sqrt{26})^2} \langle -1, 5 \rangle
\]
\[
= \frac{-3 + 20}{26} \langle -1, 5 \rangle
\]
\[
= \frac{17}{26} \langle -1, 5 \rangle
\]
\[
= \langle -\frac{17}{26}, \frac{85}{26} \rangle
\]

11. A wagon is pulled along a horizontal path by a constant force of 40 pounds. The handle of the wagon is at an angle of 30° above the horizontal. How far was the wagon pulled if the work done was \( 200\sqrt{3} \) foot pounds?

(a) 5 feet
(b) \( 5\sqrt{3} \) feet
(c) 10 feet
(d) \( 10\sqrt{3} \) feet
(e) \( 10\sqrt{3} \) feet

\[ |F| = 40 \]
\[ \theta = 30° \]

\[ W = F \cdot d = |F| |d| \cos \theta \]
\[ 200\sqrt{3} = 40 \cdot d \cos(30°) \]
\[ 200\sqrt{3} = 40 \cdot d \frac{\sqrt{3}}{2} \]
\[ 200\sqrt{3} = 20 \sqrt{3} d^2 \]
\[ d = 10 \]
12. Find the vertical and horizontal asymptotes of \( f(x) = \frac{(x-1)(x+3)}{x^2 - 1} = \frac{(x-1)(x+3)}{(x+1)(x-1)} \rightarrow \frac{x+3}{x+1} \)

(a) \( x = -1, y = 1 \)
(b) \( x = -1, x = 1, y = 1 \)
(c) \( x = -1, y = 0 \)
(d) \( x = -1, x = 1, y = 0 \)
(e) \( x = -1, x = 1, y = 1, y = -3 \)

13. Find a vector of length 4 that is in the same direction as the vector \( (3, 4) \).

(a) \( (12, 16) \)
(b) \( \left< \frac{3}{5}, \frac{4}{5} \right> \)
(c) \( \left< \frac{12}{5}, \frac{16}{5} \right> \)
(d) \( \left< \frac{16}{5}, \frac{12}{5} \right> \)
(e) \( \left< \frac{15}{4}, 5 \right> \)

\[ |a| = 5 \]
\[ \frac{4}{5} \left< 3, 4 \right> = \left< \frac{12}{5}, \frac{16}{5} \right> \]

14. Find \( \lim_{x \to 8} \frac{5x - x^2}{(5-x)^2} = \lim_{x \to 5^+} \frac{x(5-x)}{(5-x)^2} = \lim_{x \to 5^+} \frac{x}{5-x} \)

(a) \( \infty \)
(b) DNE
(c) \( 5 \)
(d) \( -\infty \)
(e) \( 1 \)

As \( x \to 5^+ \)
\( 5-x \to 0 \) from the negative side.

15. Evaluate \( \lim_{x \to -\infty} \frac{\sqrt{9x^2 + 4x}}{4x + 1} \)

(a) \( \infty \)
(b) \( \frac{-3}{4} \)
(c) \( \frac{-9}{4} \)
(d) \( \frac{3}{4} \)
(e) \( \frac{9}{4} \)

\[ = \lim_{x \to -\infty} \frac{\sqrt{9x^2 + 4x}}{4x + 1} = \lim_{x \to -\infty} \frac{\sqrt{9 + \frac{4}{x}}}{\frac{4}{x} + \frac{1}{x}} = \lim_{x \to -\infty} \frac{-\frac{3}{x}}{\frac{4}{x} + \frac{1}{x}} = \frac{-3}{4} \]
16. Evaluate \( \lim_{x \to \infty} \frac{7 - 3x^5}{x^3 + 1} \). 
(a) None of these 
(b) 0 
(c) \( \infty \) 
(d) -3 
(e) -\infty

\[ \lim_{x \to \infty} \frac{\frac{7}{x^3} - \frac{3}{x^2}}{1 + \frac{1}{x^3}} = \lim_{x \to \infty} \frac{7x - 3}{x^3} \to 0 \quad \text{and} \quad \frac{1}{x^3} \to 0 \]

\[ \text{as } x \to \infty \quad \frac{7}{x^2} \to 0 \quad \text{and} \quad \frac{1}{x^2} \to 0 \]

\[ \text{and} \quad -3x^2 \to -\infty \]

17. Consider the function \( g(x) = x^3 - 6x + 5 \). Which of these intervals must contain the solution \( g(x) = 12 \)?

(a) \([2, 3]\) 
(b) \([0, 1]\) 
(c) \([1, 2]\) 
(d) \([-2, 1]\) 
(e) \([-1, 0]\)

\[ g(2) = 8 - 12 + 5 = 1 \]
\[ g(3) = 27 - 18 + 5 = 14 \]

\( g(x) \) is continuous by IVT there is a \( c \) with \( 2 < c < 3 \) and \( g(c) = 12 \)

**PART II WORK OUT**

**Directions:** Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

18. (6 points) For \( f(x) = \begin{cases} 
 5x + 2 & \text{if } x \leq 1 \\
 8x + 1 & \text{if } 1 < x \leq 3 \\
 x^2 + 11 & \text{if } x > 3
\end{cases} \)

(a) Evaluate \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1} (3x + 1) = 4 \)

(b) Evaluate \( \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3x + 1) = 10 \)

\[ \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3x + 1) = 10 \]

\[ \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 + 11) = 20 \]

\[ \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 + 11) = 20 \]
19. (12 points) Evaluate these limits. Do not use the L'Hopital method.

(a) \[
\lim_{x \to 2} \frac{x - 2}{\sqrt{4x + 1} - 3} = \lim_{x \to 2} \frac{(\sqrt{4x + 1} + 3)}{(\sqrt{4x + 1})^2 - 9} = \lim_{x \to 2} \frac{(x - 2)(\sqrt{4x + 1} + 3)}{4x - 8}
\]

= \lim_{x \to 2} \frac{(x - 2)(\sqrt{4x + 1} + 3)}{4(x - 2)} = \lim_{x \to 2} \frac{\sqrt{4x + 1} + 3}{4} = \frac{\sqrt{9} + 3}{4} = \frac{6}{4} = \frac{3}{2}

(b) \[
\lim_{x \to -\infty} \frac{7x^2 + 2x}{(x + 1)(5x - 2)} = \lim_{x \to -\infty} \frac{7x^2 + 2x}{5x^2 + 3x - 2} \cdot \frac{1}{x^2} = \frac{7}{5}
\]

(c) \[
\lim_{x \to 6} \left[ \frac{1}{x - 5} - \frac{10}{x^2 - 25} \right] = \lim_{x \to 6} \left( \frac{x + 5}{(x + 5)(x - 5)} - \frac{10}{(x + 5)(x - 5)} \right)
\]

= \lim_{x \to 5} \frac{x + 5 - 10}{(x + 5)(x - 5)} = \lim_{x \to 5} \frac{x - 5}{(x + 5)(x - 5)} = \frac{1}{10}

\frac{1}{x + 5} = \frac{1}{10}
20. (6 points) Find the value(s) of $A$ that will make the function $g(x)$ continuous. If there are not values possible, then explain why.

$$g(x) = \begin{cases} \frac{A^2x + 9A}{Ax - 3} & \text{if } x \leq 2 \\ A^2 - 3 & \text{if } x > 2 \end{cases}$$

For $g(x)$ cont. we need $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x)$

$$\lim_{x \to 2^-} g(x) = \frac{A^2(2) + 9A}{2A - 3} = 2A^2 + 9A$$

$$\lim_{x \to 2^+} g(x) = A^2 - 3 = 2A - 3$$

$$2A^2 + 9A = 2A - 3$$

$$2A^2 + 7A + 3 = 0$$

$$(2A + 1)(A + 3)$$

$A = \frac{-1}{2}$

$A = -3$

21. (9 points) Consider the curve $x(t) = t + 3$, $y(t) = t^2 - 1$.

(a) Is the point $(8, 30)$ on the graph of the curve? Justify your answer.

If $x = 8$ then

$$\begin{align*}
8 &= t + 3 \\
5 &= t
\end{align*}$$

$y(5) = 5^2 - 1 = 24 \neq 30$

(b) Eliminate the parameter to find a cartesian equation.

$$x = t + 3$$

$$y = (x - 3)^2 - 1 = x^2 - 6x + 8$$

(c) Sketch the curve on the grid below. Be sure to include direction on the graph.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x$</th>
<th>$y$</th>
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<tbody>
<tr>
<td>-2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>0</td>
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<td>2</td>
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22. (7 pts) Find $f'(x)$ using the limit definition of the derivative for $f(x) = x^2 + 3x + 2$,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 + 3x + 2)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h + 2 - x^2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} = \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$$

$$= \lim_{h \to 0} 2x + h + 3 = 2x + 0 + 3 = 2x + 3 = f'(x)$$

23. (9 pts) Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ with magnitudes 5 lbs and 10 lbs, respectively, act on an object at a point $P$ as shown.

(a) Find the vector, $\mathbf{F}_1$.

$$\mathbf{F}_1 = \langle 0, 5 \rangle$$

(b) Find the vector, $\mathbf{F}_2$. Evaluate trig functions.

$$\mathbf{F}_2 = \langle 10 \cos 60^\circ, 10 \sin 60^\circ \rangle$$

$$= \langle 10 \left( \frac{1}{2} \right), 10 \left( \frac{\sqrt{3}}{2} \right) \rangle = \langle 5, 5\sqrt{3} \rangle$$

(c) Find the resultant vector, $\mathbf{F}$, which is the resulting force acting on the object.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \langle 5, 5\sqrt{3} - 5 \rangle$$