MATH 151, SPRING 2014
COMMON EXAM II - VERSION A

LAST NAME(print): ________________________________ FIRST NAME(print): ________________________________

INSTRUCTOR: ________________________________

SECTION NUMBER: ________________

ROW NUMBER: ________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-16), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 17-22), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR
“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
PART I: Multiple Choice. 3 points each

1. \( \lim_{x \to -\infty} \frac{3 + 2e^{-x}}{7 + 5e^{-x}} = \)
   (a) \( \frac{2}{5} \)
   (b) 0
   (c) \( \frac{5}{12} \)
   (d) none of these
   (e) \( \frac{3}{7} \)

2. \( \lim_{x \to 0} \frac{x \cos(5x)}{\sin(3x)} = \)
   (a) none of these
   (b) 5
   (c) 3
   (d) \( \frac{5}{3} \)
   (e) \( \frac{3}{5} \)

3. The radius of a circle is measured to be 3 meter with a possible error of ±0.06 m. Use differentials or linear approximation to estimate the maximum possible error in the area of the circle.
   (a) ±0.12\( \pi \)
   (b) ±0.54\( \pi \)
   (c) None of these
   (d) ±0.18\( \pi \)
   (e) ±0.36\( \pi \)

4. \( \lim_{x \to 3^-} \left( \frac{1}{3} \right)^{\frac{x}{3-x}} = \)
   (a) 1
   (b) 0
   (c) \( \infty \)
   (d) \(-\infty \)
   (e) None of these
5. Find the equation of the tangent line to the graph of \( f(x) = \frac{2x}{1+3x} \) at \( x = 1 \).

   (a) None of these
   (b) \( y - \frac{1}{2} = -\frac{1}{8}(x - 1) \)
   (c) \( y - \frac{1}{2} = -\frac{2}{8}(x - 1) \)
   (d) \( y - \frac{1}{2} = -\frac{3}{8}(x - 1) \)
   (e) \( y - \frac{1}{2} = \frac{1}{8}(x - 1) \)

6. Which of the following is the derivative of \( f(x) = \cot(x) \)?

   (a) \(-\csc^2(x)\)
   (b) \(-\csc(x) \cot(x)\)
   (c) \(\sec^2(x)\)
   (d) None of these.
   (e) \(\frac{1}{\sec^2(x)}\)

7. The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 10 cm and the width is 20 cm, what is the rate of change of the area of the rectangle?

   (a) 20 cm\(^2\)/sec
   (b) None of these
   (c) 160 cm\(^2\)/sec
   (d) \(-20 \text{ cm}^2/\text{sec}\)
   (e) 60 cm\(^2\)/sec

8. Suppose that \( f \) is a differentiable function, and let \( H \) be the function defined by \( H(x) = xf(3x) \). Find \( H'(2) \).

<table>
<thead>
<tr>
<th>x</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

   (a) 18
   (b) 12
   (c) 32
   (d) 16
   (e) None of these
9. At what point on the graph of \( f(x) = \sqrt{x} \) is the tangent line parallel to the line \( 2x - 3y = 4 \)?

(a) \( \left( \frac{4}{9}, \frac{2}{3} \right) \)
(b) \( \left( \frac{1}{9}, \frac{1}{3} \right) \)
(c) \( \left( \frac{9}{16}, \frac{3}{4} \right) \)
(d) \( \left( \frac{4}{3}, \frac{2}{\sqrt{3}} \right) \)
(e) None of these

10. Find all point(s) on the curve parametrized by \( x = t^2 - 2t - 3 \), \( y = t^3 - 3t^2 \) where the tangent line is horizontal.

(a) \((-4, -2)\)
(b) None of these.
(c) \((0, 0)\) and \((-3, 0)\)
(d) \((-3, -4)\) and \((-3, 0)\)
(e) \((0, 0)\) and \((0, -4)\)

11. Find all point(s) on the curve parametrized by \( x = t^2 - 2t - 3 \), \( y = t^3 - 3t^2 \) where the tangent line is vertical.

(a) None of these.
(b) \((-3, -4)\) and \((-3, 0)\)
(c) \((0, 0)\) and \((0, -4)\)
(d) \((0, 0)\) and \((-3, 0)\)
(e) \((-4, -2)\)

12. What is the slope of the line tangent to the curve parametrized by \( x = \sqrt{2t + 10} \), \( y = t^2 - 3t \) at the point \((4, 0)\)?

(a) \(-3\)
(b) \(5\sqrt{18}\)
(c) None of these
(d) \(12\)
(e) \(-3\sqrt{18}\)
13. The position function of an object moving along a straight line is given by \( s(t) = t^4 - 4t + 1 \), where position is measured in feet and time in seconds. Find the total distance traveled by the object during the first two seconds.

(a) 8 feet  
(b) 14 feet  
(c) 32 feet  
(d) 9 feet  
(e) None of these.

14. Find the 87th derivative of \( f(x) = 4 \sin(2x) \)

(a) \(-2^{87} \cos(2x)\)  
(b) None of these  
(c) \(-2^{89} \cos(2x)\)  
(d) \(-2^{89} \sin(2x)\)  
(e) \(-2^{57} \sin(2x)\)

15. Compute \( f^{(3)}(1) \) for the function \( f(x) = \frac{1}{x+1} \).

(a) \(-3/8\)  
(b) \(3/8\)  
(c) \(-6\)  
(d) None of these.  
(e) 6

16. Compute \( f'(x) \) for the function \( f(x) = e^{x^2} \).

(a) \( e^{x^2} \)  
(b) \( xe^{x^2-1} \)  
(c) \( 2xe^{x^2-1} \)  
(d) \( 2xe^{x^2} \)  
(e) \( e^{2x} \)
PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (20 points) Compute the derivative of these functions. Do not simplify after taking the derivative.

(a) \( y = \sqrt[3]{5} - x^4 + \pi^3 \)

(b) \( y = \frac{x^2 + 1}{(x^3 + 7x)^3} \)

(c) \( y = x \sec^4(5x) \)

(d) \( y = (x^5 + 8x)^6 \)

(e) \( y = \cos(x + e^{3x}) \)
18. (6 points) Let \( r(t) = (t \sin(t), t^3 - 7) \) be the position function for an object. Find the velocity and the acceleration functions for the object. Be sure to label your answers.

19. (6 points) Find the derivative of \( f(x) \). Be sure to explain why \( f \) is or is not differentiable at \( x = 1 \) and \( x = -1 \).

\[
f(x) = \begin{cases} 
1 - 3x & \text{if } x \leq -1 \\
x^2 - x + 2 & \text{if } -1 < x \leq 1 \\
x^4 - 3x + 3 & \text{if } x > 1
\end{cases}
\]
20. (6 points) Find \( \frac{dy}{dx} \) for the equation: \( \tan(3y^2) + 5x = 3x^2y \)

21. (6 points) Consider the function \( f(x) = \sqrt{7 - x} \).

   (a) Find the linear approximation, \( L(x) \), of \( f \) at \( a = 3 \).

   (b) Use \( L(x) \) to approximate \( \sqrt{5} \).
22. (8 points) Noah travels due north and Eddie travels due east from a common starting point. At time \( t \) (in seconds), Noah’s distance (in feet) from the starting location is \( y \) and Eddie’s distance from the starting location is \( x \). At what rate is the distance between Noah and Eddie changing after 2 seconds?

\[
y = 10 + 4t + \frac{1}{2}t^2
\]
\[
x = 7 + 4t
\]