11. (b) Two limit definitions of $f'(a)$ are $\lim_{x-a} f(x) - f(a)$ and $\lim_{h\to0} \frac{f(a+h) - f(a)}{h}$. For $f(x) = (2x + 1)^{3/2}$ and $a = 1$, these are represented by choices (iii) and (ii), respectively. The other is rubbish.

2. (a) For $x \neq 0$, $f'(x) = 5x^4 + \frac{1}{x}x^{-4/3}$. So $f'(1) = \frac{26}{3}$.

3. (a) Now $v(t) = s'(t) = \frac{(t^2+1)(1-t)(2t)}{(t^2+1)^2}$. Therefore, $v(2) = -\frac{3}{25}$ ft/s.

4. (d) The slope of the tangent is $f'(x) = 3x^2 - 4x$. So $f'(x) = -1$ yields $0 = 3x^2 - 4x + 1 = (3x - 1)(x - 1)$ whence $x = \frac{1}{3}$ or $x = 1$.

5. (e) For $x \neq \pm 1$, we have $g'(x) = \frac{1}{5} (x^2 - 1)^{-2/3} (2x)$. Now $\lim_{x \to 1} g(x) - g(1) = \lim_{x \to 1} \frac{x+1}{(x+1)^{2/3}} = \infty$ shows that $g'(1)$ does not exist. Similarly, $g'(-1)$ does not exist. Hence the domain of $g'$ is $\{x \in \mathbb{R} : x \neq \pm 1\}$.

6. (a) Now $f'(\theta) = \sec \theta \tan \theta$. So $f''(\theta) = \sec \theta \tan^2 \theta + \sec^3 \theta = \sec \theta (\sec^2 \theta + \tan^2 \theta)$.

7. (c) We have $r'(t) = \left[ \frac{(x-t)x-\pi(1-x)}{(x-t)^2}, \ 2\tan t \sec^2 t \right]$.

Thus $r'(\frac{\pi}{4}) = \left[ \frac{\pi}{(9/16)x^2}, \ 2\left(\frac{\sqrt{2}}{2}\right)^2 \right] = \left[ \frac{16}{4}, \ 4 \right]$.

8. (e) Write $f(x) = (x + 1/2)^{1/2}$. Then $f(1) = 2$ and $f'(x) = \frac{1}{2} (x + 1/2)^{-1/2} (1/2x^{-1/2})$. So $f'(1) = \frac{1}{4}$ and $L(x) = f(1) + f'(1)(x-1) = 2 + \frac{1}{4} (x-1)$. That is, $L(x) = \frac{1}{8}x + \frac{15}{8} = \frac{1}{8} (x + 15)$.

9. (b) The first four derivatives of $f(x) = x^{-1}$ are $-x^{-2}, 2x^{-3}, -6x^{-4}, 24x^{-5}$. So $f^{(n)}(x) = (-1)^n n!$. Therefore, $f^{(31)}(x) = -3!1/x^{32}$.

10. (c) Direct substitution yields $\lim_{\theta \to \pi/6} \frac{(1 + \cot \theta)^2}{1 - \csc \theta} = \frac{(1 + \sqrt{3})^2}{2} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$.

11. (a) As $x \to -\frac{\pi}{2}^+$, we have $\tan x = \frac{\sin x}{\cos x} \to \frac{1}{0} = -\infty$, whence $3\sin x \to 0$.

12. (a) With $F(x) = f(x^2 - 1) - f(1 - x^2)$, Chain Rule gives $F'(x) = f'(x^2 - 1)(2x) - f'(1 - x^2)(-2x)$. So $F'(1) = 4f'(0)$.

13. (d) The so-called normal line $y = \frac{x+3}{2}$ has slope $m = \frac{1}{7}$. So the slope of the tangent line at $P(1,f(1))$ is $f'(1) = -1/m = -7$. With $g(x) = x^2 f(x)$, we have $g'(x) = 2xf'(x) + x^2 f'(x)$. Since $P$ is on the normal line, $f(1) = \frac{1+1}{2} = 2$. Thus $g'(1) = 2f(1) = f'(1)$ equals $4 - 2 = 2$.

14. The square of the distance between $P(x,x^2)$ and $O(0,0)$ is $z = \sqrt{(x-0)^2 + (x^2-0)^2} = x^2 + x^4$.

When $y = x^2 = 9$ in the first quadrant, we have $x = 3$. Recall $dx/dt = 2$. Via related rates,

\[
\frac{dz}{dt} = 2x \ \frac{dx}{dt} + 4x^3 \ \frac{dx}{dt}
\]

$\frac{dz}{dt}|_{data} = 2(3)(2) + 4(3)^3(2) = 228$.

15. Recall $x = (t+9)^{1/2}$ and $y = te^{-t}$ for $-3 \leq t \leq 3$.

(a) We have $\frac{dy}{dt} = (1)e^{-t} + t(-e^{-t}) = (1-t)e^{-t}$ and $\frac{dx}{dt} = \frac{1}{2}(t+9)^{-1/2} = \frac{1}{2\sqrt{t+9}}$.

(b) Thus $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 2(1-t)\sqrt{t+9}e^{-t} = m$.

(c) For a horizontal tangent, $\frac{dy}{dx} = 0$, whence $t = 1$ and $(x,y) = (\sqrt{10}, e^{-1})$.

(d) When the curve crosses the x-axis, $y = te^{-t} = 0$, so $t = 0$ and $x = \sqrt{t+9} = 3$. From (b), the slope of the tangent line at $(3,0)$ is $m = 6$ since $t = 0$. The point-slope formula gives $y-0 = 6(x-3)$ or $y = 6x-18$ for the tangent line.

16. Recall $\sin(xy) + x\cos \left(\frac{y}{x}\right) + y = \pi$. Differentiate implicitly with respect to $x$. Substitute $(x,y) = (1,\pi)$ and $y' = 0$ where $m = y'|_{(1,\pi)}$. Finally, solve for $m$.

$\cos(xy) \cdot (1y' + xy') + x \left(-\sin \left(\frac{y}{x}\right) \cdot \frac{1}{x} y' \right) + y' = 0$

$-1(\pi + m) + 0 - \frac{1}{2}m + m = 0$

$\pi - \frac{1}{2}m = 0$

$m = -2\pi$

17. The linear approximation gives the tangent line to $f$ at $(2,3)$. So $f(2) = 3$ and $L(2) = 3$.

(a) Now $L(x) = 3x + B$. So $3 = L(2) = 6 + B$, whence $B = -3$.

(b) So $L(x) = 3x - 3$ and $L'(x) \equiv 3$. Hence $f''(2) = L'(2) = 3$. Given $f''(x) = 6$, the quadratic approximation to $f$ near $a = 2$ is

\[
Q(x) = f(2) + f'(2)(x - 2) + \frac{1}{2} f''(2)(x-2)^2
\]

$= 3 + 3(x-2) + 3(x-2)^2$

or $3x^2 - 9x + 9$. 