1. Find the points \((x, y)\) on the curve \(y = x^3 - x^2 - x + 1\) where the tangent line is horizontal.

   - A horizontal tangent line has slope zero.
   
   \[
   y' = 3x^2 - 2x - 1 = 0
   \]
   
   \[
   (3x + 1)(x - 1) = 0
   \]
   
   - Thus \(x = -\frac{1}{3}\) or \(x = 1\).
   
   - Computing respective \(y\) values yields two points: \((-\frac{1}{3}, \frac{32}{27})\) and \((1, 0)\).

2. A tank holds 5000 L of water, which drains from the bottom of the tank in 40 min. The volume of water in the tank after \(t\) minutes is

   \[
   V = 5000 \left(1 - \frac{t}{40}\right)^2, \quad 0 \leq t \leq 40.
   \]

   Find the rate at which the volume of water is changing after 5 min.

   - The rate of change of volume with respect to time is
     \[
     \frac{dV}{dt} = 10000 \left(1 - \frac{t}{40}\right) \left(-\frac{1}{40}\right).
     \]
   
   - When \(t = 5\) min, we have
     \[
     \left|\frac{dV}{dt}\right|_{t=5} = -\frac{875}{4} = -218.75 \text{ L/min}.
     \]

3. Find an equation of the tangent line to the curve \(y = \tan x\) at the point \((\pi/4, 1)\). Show your steps.

   - Compute the slope of the tangent line at \(x = \pi/4\).
     
     \[
     y' = \sec^2 x
     \]
     
     \[
     y' \left(\frac{\pi}{4}\right) = \left(\sqrt{2}\right)^2 = 2
     \]
   
   - Now use the point-slope formula.
     
     \[
     y - 1 = 2 \left(x - \frac{\pi}{4}\right)
     \]
     
     \[
     y = 2x + 1 - \frac{\pi}{2}
     \]

4. If \(f\) and \(g\) are the functions whose graphs are shown below, let \(u(x) = f(g(x))\). Find \(u'(1)\) via Chain Rule. [Graphs are piecewise linear; slopes (derivatives) may be found via rise/run.

   - To use the Chain Rule, determine requisite function values and derivatives from the plot.
     
     \[
     u'(1) = f'(g(1))g'(1) = f'(3)g'(1) = \left(-\frac{1}{4}\right)(-3) = \frac{3}{4}
     \]

5. Via implicit differentiation, find all points \((x, y)\) on the curve \(x^2y^2 + xy = 2\) where the slope of the tangent line equals \(-1\).

   - Compute \(y'\) and set it equal to \(-1\).
     
     \[
     2xy^2 + 2x^2yy' + (1)y + xy' = 0
     \]
     
     \[
     (2x^2y + x)y' = - (y + 2xy^2)
     \]
     
     \[
     y' = \frac{y(1 + 2xy)}{x(1 + 2xy)}
     \]
     
     [Use impDif to get here!] \(y' = -\frac{y}{x} = -1\)
   
   - So \(y/x = 1\) or \(y = x\). Substitute into the curve’s equation, \(x^2y^2 + xy = 2\). Then solve for \(x\) and hence \(y\) since \(y = x\). (Use Solve if desired.)
     
     \[
     x^4 + x^2 = 2
     \]
     
     \[
     x^4 + x^2 - 2 = 0
     \]
     
     \[
     (x^2 - 1)(x^2 + 2) = 0
     \]
     
     \[
     (x + 1)(x - 1)(x^2 + 2) = 0
     \]
     
     Thus \(x = -1\) or \(x = 1\) (plus two complex solutions, \(x = \pm\sqrt{2}i\), which we toss out).
   
   - Hence our two points are \((-1, -1)\) and \((1, 1)\).
6. Find the tangent vector to the curve 
   \[ r(t) = \left[ t + 2 \sin t \quad t + \sin 3t \right] \] for \( t = 0 \).

   - Compute \( r'(0) \).
   
   \[ r'(t) = \left[ 1 + 2\cos t \quad 1 + 3\cos 3t \right] \]
   \[ r'(0) = \left[ 3 \quad 4 \right] \]

7. Let \( f(x) = 3x^5 - 10x^3 + 5 \). Find the intervals on which \( f''(x) > 0 \).

   - Compute derivatives, then solve an inequality.
     \( f'(x) = 15x^4 - 30x^2 \) and \( f''(x) = 60x^3 - 60x \)
     \[ f''(x) = 60x(x - 1)(x + 1) > 0 \]

   - A sign analysis (or your calculator’s Solve command) yields
     \[-1 < x < 0 \quad \text{or} \quad x > 1.\]

8. The parametric curve \( x = (t^2 - 3) \), \( y = 3(t^2 - 3) \) crosses itself at the origin. Find equations of both tangent lines there.

   - For \((x, y) = (0, 0)\), we have \( x = (t^2 - 3) \) \( t = 0 \)
     and \( y = 3(t^2 - 3) = 0 \). Solving these two equations yields \( t = -\sqrt{3} \) or \( t = \sqrt{3} \).
   
   - Compute the slope of a tangent line in general.
     \[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{3t^2 - 3} \]

   - For \( t = -\sqrt{3} \), \( m = -\sqrt{3} \), whereas for \( t = \sqrt{3} \), \( m = \sqrt{3} \).
   
   - Since lines through the origin have the form \( y = mx \), the tangent lines in question are
     \( y = -\sqrt{3}x \) and \( y = \sqrt{3}x \).

9. The volume of a cube is increasing at 10 cm\(^3\)/min. How fast is the surface area increasing when the length of an edge is 30 cm?

   - Let \( x \) be the length of an edge of the cube. Then the cube’s surface area and volume are \( S = 6x^2 \) and \( V = x^3 \), respectively.
   
   - Thus \( x = V^{1/3} \) and hence \( S = 6V^{2/3} \). Via related rates, we have
     \[ \frac{dS}{dt} = 4V^{-1/3} \frac{dV}{dt} = \frac{4(dV/dt)}{\sqrt[3]{V}}. \]

   - When \( x = 30 \), \( V = 30^3 \), and given \( dV/dt = 10 \), we see that at this instant
     \[ \frac{dS}{dt} = \frac{40}{30} = \frac{4}{3} \text{ cm}^2/\text{min}. \]

10. Let \( f \) be a function such that \( f(1) = 2 \) and whose derivative is \( f'(x) = \sqrt{x^3 + 1} \).

   - (a) Use the linear approximation to \( f \) at \( a = 1 \) to approximate \( f(1.1) \) to 4 decimal places.
     - The linear approximation is
       \[ L(x) = f(a) + f'(a)(x-a) \]
       \[ L(x) = f(1) + f'(1)(x-1) \]
       \[ L(x) = 2 + \sqrt[3]{2}(x-1) \]
       \[ f(1.1) \approx L(1.1) = 2 + \sqrt[3]{2}(0.1) \approx 2.1414. \]

   - (b) [BONUS] Use the quadratic approximation to \( f \) at \( a = 1 \) similarly approximate \( f(1.1) \).
     - Now \( f''(x) = \frac{1}{2}(x^3 + 1)^{-1/2}(3x^2) \). The quadratic approximation is
       \[ Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 \]
       \[ Q(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 \]
       \[ Q(x) = 2 + \sqrt[3]{2}(x-1) + \frac{3}{4\sqrt[3]{2}}(x-1)^2 \]
       \[ Q(1.1) \approx 2 + \sqrt[3]{2}(0.1) + \frac{3}{4\sqrt[3]{2}}(0.1)^2 \approx 2.1467. \]

11. Find an equation of the tangent line to the curve \( 2e^{xy} = x+y \) at \( P(0, 2) \).

   - Implicit differentiation gives
     \[ 2e^{xy}((1)y + xy') = 1 + y' \]
     \[ [at P:] \quad 2(2 + 0) = 1 + y'(P) \]
     \[ m = y'(P) = 3. \]

   - Now use the point-slope formula.
     \[ y - 2 = 3(x - 0) \]
     \[ y = 3x + 2 \]

12. Use a theorem to find \( g'(2) \) where \( g \) is the inverse function of \( f(x) = \sqrt[3]{x^3 + x^2 + x + 1} \).

   - By inspection, we see \( f \) maps 1 to \( \sqrt[3]{4} = 2 \).
     That is \( f(1) = 2 \), whence \( g(2) = 1 \). Note that
     \[ f'(x) = \frac{1}{2}(x^3 + x^2 + x + 1)^{-1/2}(3x^2 + 2x + 1). \]
   
   - A theorem therefore yields
     \[ g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{1}{6/4} = \frac{4}{6} = \frac{2}{3}. \]
13. Find the domain and range of \( f(t) = \sqrt{t} \ln(t^2 - 1) \).

- We require \( t \geq 0 \) and \( t^2 - 1 > 0 \). The second inequality implies \( t^2 > 1 \) or \(|t| > 1\). Since \( t \) must be positive, the domain is \( t > 1 \).
- Now \( f \) is continuous on its domain. Moreover, as \( t \to 1^+ \), \( f(t) = \sqrt{t} \ln(t^2 - 1) \to -\infty \). Also, as \( t \to \infty \), \( f(t) = \sqrt{t} \ln(t^2 - 1) \to \infty \).
- Therefore, the range of \( f \) is \( \mathbb{R} = (-\infty, \infty) \), the set of all real numbers.

14. Use logarithmic differentiation to find the derivative of \( y = \frac{(x^3 + 1)^4 \sin^2 x}{x^{1/3}} \) by hand.

\[
\ln y = \frac{4\ln(x^3 + 1) + 2\ln(\sin x) - \frac{1}{3}\ln x}{x^{1/3}}
\]

\[
\frac{1}{y} \frac{dy}{dx} = \frac{4(3x^2) + 2\cos x}{x^3 + 1} \frac{1}{\sin x} - \frac{1}{3x}
\]

\[
\frac{dy}{dx} = \left( \frac{(x^3 + 1)^4 \sin^2 x}{x^{1/3}} \right) \left( \frac{12x^2 \sin^2 x + 2\cot x - \frac{1}{3x}}{x^3 + 1} \right)
\]