Last name, First name (print): ________________________________

Signature: _______________________________________

Instructor’s name: ________________________________

Section No: ________________________________

GUIDELINES

• In Part 1 (Problems 1–13), mark your responses on your ScanTron form using a No: 2 pencil. For your own record, mark your choices on the examination paper as well. ScanTrons will be collected at the conclusion of the examination; they will not be returned.

• In Part 2 (Problems 14–17), present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

• Be sure to write your name, section number, and version letter of the examination on the ScanTron form.

• Calculators should not be used throughout the examination. Mobile phones must be switched off. Examinees found using a mobile phone during the course of the test are subject to being ejected from the examination hall, and to being given a zero on the examination.
PART 1 (52 points)
Each question is worth 4 points. Mark your responses on the ScanTron form and on the examination paper itself.

1. Calculate the value of sin(\tan^{-1}(1/2)).
   (a) $1/\sqrt{5}$
   (b) $2/\sqrt{5}$
   (c) $1/5$
   (d) $2/5$
   (e) $1/\sqrt{3}$

2. Given that $2\pi \leq \theta \leq 3\pi$, determine the value of $\cos^{-1}(\cos \theta)$.
   (a) $\pi - \theta$
   (b) $2\pi - \theta$
   (c) $\theta$
   (d) $\theta - \pi$
   (e) $\theta - 2\pi$

3. Compute $\lim_{x \to 0^+} \frac{\ln(x)}{x}$.
   (a) $+\infty$
   (b) $-\infty$
   (c) 0
   (d) $-1$
   (e) 1

4. Calculate $\lim_{x \to 0} \frac{x^3}{\tan^{-1}(x) - x}$.
   (a) 6
   (b) 3
   (c) 0
   (d) $-6$
   (e) $-3$
5. Given that \( f'(x) = \frac{x^2 + 1}{x^2} \) for \( x > 0 \), and that \( f(1) = 1 \), determine \( f(x) \) for \( x > 0 \).

(a) \( 3 - \frac{2}{x^3} \)
(b) \( 1 + x - \frac{1}{x} \)
(c) \( 2 - \frac{1}{x} \)
(d) \( \frac{x^3 + 3x}{x^3} - 3 \)
(e) \( 2 + x - \frac{2}{x^3} \)

6. Consider the function \( f \) defined by \( f(x) = xe^{-2x^2} \), for \( -\infty < x < \infty \). Determine the interval(s) in which \( f \) is increasing.

(a) \((-1/2, 1/2)\)
(b) \((-\infty, \infty)\)
(c) \((-1/4, 1/4)\)
(d) \((-\infty, -1/2)\) and \((1/2, \infty)\)
(e) \((-\infty, -1/4)\) and \((1/4, \infty)\)

7. Let \( f(x) = \frac{x^4}{4} + \frac{x^3}{3} \), for \(-2 \leq x \leq 2\). Determine the value of \( x \) at which \( f \) attains its absolute (global) minimum in the closed interval \([-2, 2]\).

(a) \( x = -2 \)
(b) \( x = -1 \)
(c) \( x = 2 \)
(d) \( x = 1 \)
(e) \( x = 0 \)
8. Consider the function \( f \) defined by \( f(x) = \frac{x^2}{2} + 2\cos x \), for \( 0 \leq x \leq \pi \). Which of the following statements is true?

(a) The graph of \( f \) is concave upwards throughout the interval \((0, \pi)\).

(b) The graph of \( f \) is concave upwards in the interval \((0, \pi/3)\).

(c) The graph of \( f \) is concave downwards in the interval \((0, \pi/3)\).

(d) The graph of \( f \) is concave downwards throughout the interval \((0, \pi)\).

(e) The graph of \( f \) is concave downwards in the interval \((\pi/3, \pi)\).

9. Let \( f(x) = \frac{x^6}{30} - \frac{x^4}{12} + x - 1 \), for \( -\infty < x < \infty \). Determine the \( x \) co-ordinates of all the inflection points on the graph of \( f \).

(a) only \( x = 1 \)

(b) \( x = -1, 0, 1 \)

(c) only \( x = -1 \)

(d) only \( x = -1, 1 \)

(e) only \( x = 0 \)

10. Given that \( f(x) = (\tan x)^{\sin x} \), for \( 0 < x < \pi/2 \), compute \( f'(\pi/4) \).

(a) \( \sqrt{2} \)

(b) \( 2\sqrt{2} \)

(c) \( \sqrt{2}/2 \)

(d) \( 2\sqrt{2} \)

(e) 2
11. Suppose that the function $f$ is continuous in an open interval $I$, and that $p$ is a point in $I$. Which of the following statements is/are true?

(i) If $f$ is not differentiable at $p$, then $f$ must have a local maximum or local minimum at $p$.
(ii) If $f$ is differentiable at $p$ and $f'(p) = 0$, then $f$ must have a local maximum or local minimum at $p$.
(iii) If $f$ has a local maximum or local minimum at $p$, then $f$ must be differentiable at $p$ and $f'(p)$ must equal zero.
(iv) If $f$ is differentiable at $p$ and $f'(p) \neq 0$, then $f$ cannot have a local maximum or a local minimum at $p$.

(a) only (iii) and (iv)
(b) only (i) and (iv)
(c) only (ii) and (iv)
(d) only (iii)
(e) only (iv)

12. Let $f$ be the one-to-one function defined by $f(x) = \sec^2(x)$, for $0 \leq x < \pi/2$; let $g$ denote the inverse of $f$. Compute the value of $g'(4/3)$.

(a) $3/4$
(b) $\sqrt{3}/8$
(c) $8/(3\sqrt{3})$
(d) $(3\sqrt{3})/8$
(e) $8/\sqrt{3}$

13. Suppose that a straight line passes through the origin, and that it is a tangent to the curve $y = \ln x$ at some point $(a, \ln a)$, $a > 0$. What is the slope of the line?

(a) 1
(b) $e$
(c) $1/e$
(d) $e^2$
(e) $1/e^2$
14. A bacterial culture starts with 50 bacteria and after 2 hours there are 400 bacteria. Assume that the culture grows at a rate proportional to its size, i.e., it exhibits exponential growth.

(i) \(7\) points) Find an expression for the population after \(t\) hours.

(ii) \(5\) points) When will the population reach 100,000?
15. (12 points) A rectangular storage container with a closed top is to have a volume of 10 cubic meters. The length of its base is twice the width. Material for the base costs 10 dollars per square meter, whereas material for the sides and the top costs 6 dollars per square meter. Determine the width that minimizes the cost of the container. (Your analysis must include showing that the final answer does, in fact, produce the required minimum.)
16. (12 points) Compute

\[ \lim_{x \to \infty} \left( 1 + x + x^2 \right)^{\frac{1}{x}}. \]
17. Consider the function $H$ defined by

$$H(x) = \sin^{-1}\left(2e^x - \frac{5}{2}\right).$$

(i) (4 points) Determine the domain of $H$. Write the answer using interval notation.

(ii) (8 points) Obtain an equation of the tangent line to the graph of $H$, at the point $(0, H(0))$. 


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