MATH 151, SPRING 2015
COMMON EXAM II - VERSION B

LAST NAME: ___________________________ FIRST NAME: ___________________________

INSTRUCTOR: __________________________

SECTION NUMBER: _________________

UIN: ___________________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ___________________________

DO NOT WRITE BELOW!

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100
PART I: Multiple Choice. 3 points each

1. Find the slope of the tangent line to the graph of \( x^3 + y^3 = 6xy - 1 \) at the point \((2, 3)\).
   
   (a) \( \frac{6}{21} \)
   
   (b) \( -\frac{12}{21} \)
   
   (c) \( -\frac{4}{3} \)
   
   (d) \( \frac{4}{5} \)
   
   (e) \( \frac{2}{5} \)

2. \( \lim_{t \to \infty} \frac{8}{1 + e^{-0.25t}} = \)
   
   (a) 4
   
   (b) 2
   
   (c) 8
   
   (d) 0
   
   (e) \( \infty \)

3. \( \lim_{x \to 3^+} \left( \frac{1}{4} \right) \frac{x}{x - 3} = \)
   
   (a) \( \frac{1}{4} \)
   
   (b) \( \infty \)
   
   (c) \( -\infty \)
   
   (d) 0
   
   (e) 4
4. A diver jumps off a diving board that is 16 feet above the water. The height of the diver at time \( t \) seconds is given to be \( s(t) = -8t^2 + 8t + 16 \). What is the velocity of the diver when he hits the water?

(a) \(-32 \text{ ft/s}\)
(b) \(-24 \text{ ft/s}\)
(c) \(-8 \text{ ft/s}\)
(d) \(-16 \text{ ft/s}\)
(e) \(0 \text{ ft/s}\)

5. Find the slope of the tangent line to the curve \( x = t^2 - 4, y = \frac{t}{2} \) at the point \((12, 2)\).

(a) \(\frac{1}{16}\)
(b) 16
(c) \(\frac{1}{48}\)
(d) \(\frac{1}{2}\)
(e) 48

6. If \( H(x) = x^3 + g(f(x)) \), find \( H'(-2) \) given that \( f(-2) = 5, g'(5) = 3, f'(-2) = 2, g(5) = 7, g'(-2) = 4 \) and \( g(-2) = -1 \).

(a) 30
(b) 18
(c) 0
(d) 15
(e) \(-6\)
7. \( \lim_{\theta \to 0} \frac{\sin^3(2\theta)}{\theta^3} = \)

(a) 8
(b) \( \frac{1}{8} \)
(c) 2
(d) \( \frac{1}{2} \)
(e) The limit does not exist

8. Find \( f''\left(\frac{\pi}{6}\right) \) if \( f(x) = e^\sin x \).

(a) \( \sqrt{e} \)
(b) \( \frac{5\sqrt{e}}{4} \)
(c) \( -\frac{\sqrt{3}\sqrt{e}}{2} \)
(d) \( \frac{\sqrt{e}}{4} \)
(e) \( -\frac{\sqrt{e}}{4} \)

9. If \( f(x) = e^x(x^2 - 2x + 2) \), find the equation of the tangent line at \( x = 1 \).

(a) \( y = ex - e \)
(b) \( y = ex + e \)
(c) \( y = 3ex - 2e \)
(d) \( y = ex \)
(e) \( y = 5ex - 4e \)
10. If $g$ is the inverse of $f$, find $g'(3)$ if $f(x) = \frac{1}{4}x^3 + x - 1$.

(a) $\frac{4}{31}$
(b) $\frac{1}{4}$
(c) $\frac{1}{3}$
(d) $\frac{4}{7}$
(e) 4

11. Find the point of intersection of the curves $r_1(t) = \langle 1 - t, 3 + t^2 \rangle$ and $r_2(w) = \langle w - 2, w^2 \rangle$.

(a) $(1, 2)$
(b) $(0, 2)$
(c) $(0, 4)$
(d) $(1, 1)$
(e) $(-1, 1)$

12. If $xy = 4$, find $\frac{dy}{dt}$ when $x = 8$ given that $\frac{dx}{dt} = 10$.

(a) $-\frac{5}{8}$
(b) $-\frac{1}{8}$
(c) $\frac{5}{8}$
(d) $\frac{1}{8}$
(e) $-\frac{9}{8}$
13. Find the quadratic approximation of \( f(x) = \tan x \) at \( x = \frac{\pi}{4} \).

(a) \( Q(x) = 1 + 2 \left( x - \frac{\pi}{4} \right) + 4 \left( x - \frac{\pi}{4} \right)^2 \)

(b) \( Q(x) = 1 + \frac{1}{2} \left( x - \frac{\pi}{4} \right) + \frac{1}{2} \left( x - \frac{\pi}{4} \right)^2 \)

(c) \( Q(x) = 1 + \frac{1}{2} \left( x - \frac{\pi}{4} \right) + \frac{1}{4} \left( x - \frac{\pi}{4} \right)^2 \)

(d) \( Q(x) = 1 + 2 \left( x - \frac{\pi}{4} \right) + \left( x - \frac{\pi}{4} \right)^2 \)

(e) \( Q(x) = 1 + 2 \left( x - \frac{\pi}{4} \right) + 2 \left( x - \frac{\pi}{4} \right)^2 \)

14. Find \( \frac{dy}{dx} \) at the point (1, 0) if \( (\sin(\pi x) + \cos(\pi y))^3 = x^2 y \).

(a) \(-3\)

(b) \(3\pi\)

(c) \(3\)

(d) \(\frac{3}{\pi}\)

(e) \(-3\pi\)

15. Using differentials or a linear approximation, estimate \((2.01)^4\).

(a) 16.32

(b) 16.26

(c) 16.12

(d) 16.8

(e) 16.48
PART II: Work Out

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. A ladder 10 feet long is leaning against a vertical wall. The base of the ladder is pulled away from the wall at a rate of 4 ft/s.

![Diagram of a ladder leaning against a wall](image)

a) (6 pts) How fast is the top of the ladder sliding down the wall when the base is 6 feet from the wall?

b) (5 pts) Find the rate at which angle between the ladder and the wall is changing when the base of the ladder is 6 feet from the wall.
17. \( f(x) = \begin{cases} 
bx^2 + 3x + 2 & \text{if } x \leq -1 \\
ax + 5 & \text{if } x > -1 
\end{cases} \)

a) (6 pts) Find the values of \( a \) and \( b \) that make \( f(x) \) differentiable everywhere.

b) (5 pts) For \( a \) and \( b \) found above, what is \( f'(x) \)?
18. a) (3 pts) Show by means of a sketch that there are two lines tangent to the parabola $y = 2x^2$ that pass through the point $(1, -6)$. 

b) (8 pts) Find an equation of each of these tangent lines.
19. Consider the vector function \( \mathbf{r}(t) = \left( \sqrt{4 - t^2}, \sin\left(\frac{1}{t}\right) \right) \).

a) (4 pts) Find the domain of \( \mathbf{r}(t) \). Express your answer in interval notation.

b) (7 pts) Find \( \mathbf{r}'(t) \) and the domain of \( \mathbf{r}'(t) \).

20. Given that \( x = t^3 + 3t^2 \) and \( y = t^2 - 5t \):

a.) (6 pts) Find all point(s) on the curve where the tangent line is vertical.

b.) (5 pts) Find all point(s) on the curve where the tangent line is horizontal.