1. (D) With \( f(x) = e^{\sin x} \), we have \( f'(x) = e^{\sin x} \cos x \) and \( f''(x) = e^{\sin x} \cos^2 x - e^{\sin x} \sin x \). Thus \( f''(\frac{\pi}{6}) = \sqrt{6}/4 \).

2. (B) With \( H(x) = x^3 + g(f(x)) \), we have:
   \[ H'(-2) = 3(-2)^2 + g'(f(-2))f'(-2) = 12 + g'(5) \cdot 2 = 12 + 3(2) = 18. \]

3. (A) As \( \theta \to 0 \), \( \frac{\sin^2(\theta)}{\theta^2} = \frac{8}{\theta^2} \to 8(1)^3 = 8. \)

4. (E) If \( x^2 + y^2 = 6xy - 1 \), then \( 3x^2 + 3y^2y' = 6(y + xy') \) or \( x^2 + y^2y' = 2y + 2xy' \). So \( y' = \frac{4x^2 - 2x}{6y - 4} = \frac{2}{3} \) at \((x,y) = (2,3)\).

5. (A) With \( x = t^2 - 4 \) and \( y = t/2 \), the point \((12,2)\) corresponds to \( t = 4 \). The slope of the tangent line is \( \frac{dy}{dt}/\frac{dx}{dt} = \frac{1/2}{2t} = \frac{1}{4} \) at \( t = 2 \).

6. (B) When the diver hits the water his position (height) is zero. Solve \( s(t) = -8t^2 + 8t + 16 = 0 \) for \( t > 0 \) to obtain \( t = 2 \). At this instant, the velocity is \( s'(2) \) or \(-16t+8 \big|_{t=2} = -24 \) ft/s.

7. (E) With \( f(x) = \tan x \), we have \( f'(x) = \sec^2 x \) and \( f''(x) = 2\sec^2 x \tan x \). Thus \( f'\left(\frac{\pi}{4}\right) = 1, f''\left(\frac{\pi}{4}\right) = 2 \) and \( f''\left(\frac{\pi}{4}\right) = 4 \). The quadratic approximation of \( f \) at \( \frac{\pi}{4} \) is \( Q(x) = f\left(\frac{\pi}{4}\right) + f'(\frac{\pi}{4}) \left( x - \frac{\pi}{4} \right) + \frac{1}{2} f''(\frac{\pi}{4}) \left( x - \frac{\pi}{4} \right)^2 \) or \( Q(x) = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 \).

8. (D) With \( f(x) = (x^2 - 2x + 2)e^x \), we have \( f(1) = e \) and \( f'(1) = \left((2x-2)e^x + (x^2-2x+2)e^x\right)\big|_{x=1} = e \). The tangent line is \( y - e = e(x - 1) \) or \( y = ex \).

9. (C) As \( t \to 0 \), \( \frac{\sqrt{1 + e^{-20t^2}}}{8} \to 8 \) at \( t = 0 \).

10. (A) With \( f(x) = x^4 \), we have \( f'(x) = 4x^3 \). Therefore, \( f(2) = 16 + f'(2) = 32 \). The linear approximation is \( L(x) = f(2) + f'(2)(x - 2) = 16 + 32(x - 2) \). So \( (2.01)^4 \approx f(2.01) \approx L(2.01) = 16.32 \).

11. (B) With \( f(x) = \frac{1}{2}x^3 + x - 1 \), we have \( f(2) = 3 \). Thus \( g(3) = f^{-1}(3) = 2 \). So \( g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(2)} \). Since \( f'(x) = \frac{3}{2}x^2 + 1 \), we have \( f''(2) = 4 \) and \( g''(3) = \frac{1}{4} \).

12. (D) As \( x \to 3^+ \), \( \frac{x - 3}{\sqrt{x - 3}} \to \left(\frac{3}{2}\right)^{3/2} - \frac{3}{2} \).

13. (C) At the intersection we have \( r_1(t) = r_2(w) \) whence \( 1-t = w - 2 \) and \( 3 + w^2 = w^2 \). So \( w = 3 - t \). Hence \( 3 + t^2 = 9 - 6t + 2t^2 \) or \( 6t = 6 \). So \( t = 1 \) and \( w = 2 \). Thus \( r_1(1) = r_2(2) = [0,4]; intersection point: (0,4) \).

14. (A) If \( xy = 4 \), then \( \frac{dy}{dt} y + x \frac{dy}{dt} = 0 \). When \( x = 8, y = \frac{1}{2} \).

15. (X) The problem is ill-posed. The point \((x,y) = (1,0)\) is not on the curve \((\sin \pi x + \cos \pi y)^2 = x^2y \). lest \( 1 = 0 \) via substitution. All students were given full credit on this problem.

16. (a) From the diagram, \( x^2 + y^2 = 10^2 \). Thus \( 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \) or \( \frac{dy}{dt} = \frac{-x \frac{dx}{dt}}{y} \). When \( x = 6 \), \( y = \sqrt{100 - 36} = 8 \). Recall \( \frac{dx}{dt} = 3 \). Hence \( \frac{dy}{dt} = -\frac{6 \cdot (3)}{8} = \frac{-9}{4} \) ft/s.

(b) Now \( \sin \theta = \frac{5}{10} \), so \( \cos \theta \cdot \frac{dy}{dt} = \frac{dx}{dt} \) or \( \frac{dx}{dt} = \frac{dy}{dt} = \frac{dx}{dt} \) or \( \cos \theta \cdot \frac{dy}{dt} = \frac{dx}{dt} \).

(c) Recalling the equation of a circle, \( \cos \theta \cdot \frac{dy}{dt} = \frac{dx}{dt} \).

17. Recall \( f(x) = \begin{cases} bx^2 + 2x + 1 & \text{if } x \leq -1; \\ ax + 3 & \text{if } x > -1. \end{cases} \)

(a/b) As \( f \) is piecewise polynomial, it is differentiable for \( x \neq -1 \). Make \( f \) differentiable (and therefore also continuous) at \( x = -1 \).

• Equate left and right limits: \( b - 1 = 3 - a \), whence \( a = 4 - b \).

• Equate left and right slopes: \( -2b + 2 = a \).

• Thus \( 4 - b = -2b + 2 \), so \( b = -2, a = 6 \).

Therefore, \( f'(x) = \begin{cases} 2 - 4x & \text{if } x \leq -1; \\ 6 & \text{if } x > -1. \end{cases} \)

18. (a) Consider parabola \( y = f(x) = 2x^2 \) and point \( P(1,-6) \). Line \( y = -6 \) contains \( P \). Rotate about \( P \) so it touches parabola: two tangent lines! (Pic @ #17.)

(b) The tangent line at \( a \) is \( m = f'(a) \) \( x - a = 2ax - 2a^2 \). Suppose it contains \( P(1,-6) \). This implies \( -6 = 4a - 2a^2 \). Hence \( a = -1 \) or \( a = 3 \), which give \( y = -4x - 2 \) or \( y = 12x - 18 \).

19. \( r(t) = \begin{bmatrix} \sqrt{9 - t^2} \end{bmatrix}, \sin \left(\frac{1}{t}\right) \end{bmatrix} = \begin{bmatrix} (9 - t^2)^{1/2} \sin \left(\frac{1}{t}\right) \end{bmatrix} \).

(a) We need \( |t| \leq 3 \) and \( t \neq 0 \) for \( r(t) \) to be defined.

(b) The domain of \( r \) is \([-3,0) \cup (0,3) \).

(b) Now \( \frac{dr}{dt} = \begin{bmatrix} \frac{-t}{\sqrt{9 - t^2}} \end{bmatrix}, \frac{t^2}{\sqrt{9 - t^2}} \end{bmatrix}, \frac{t}{\sqrt{9 - t^2}} \end{bmatrix} \) or \( \left[ \begin{bmatrix} \cos \left(1/\sqrt{t^2} \end{bmatrix}, \frac{t^2}{\sqrt{9 - t^2}} \end{bmatrix} \right] \end{bmatrix} \]. Domain of \( \frac{dr}{dt} \): \((-3,0) \cup (0,3) \).

20. (a) With \( x = t^3 + 3t^2 \), solve \( \frac{dx}{dt} = 3t^2 + 6t = 0 \) to obtain \( t = -2 \) and \( t = 0 \). (With \( y = t^2 - 2t \), note that \( \frac{dy}{dt} = 2t = 2 \neq 0 \) thereat.) Thus vertical tangent lines occur at \((4,8)\) and \((0,0)\), respectively.

(b) Solve \( \frac{dy}{dt} = 0 \) to get \( t = 1 \). (Note that \( \frac{dy}{dt} \neq 0 \) thereat.) A horizontal tangent line occurs at \((4,-1)\).
1. (E) If $x^2 + y^3 = 6xy - 1$, then $3x^2 + 3y^2y' = 6(y + xy')$ or $x^2 + y^2y' = 2y + 2xy'$. So $y' = \frac{2y - x^2}{3y - 2x}$ or $\frac{6 - 4y}{3} = \frac{3}{2}$ at $(x,y) = (2,3)$.

2. (C) As $t \to \infty$, $\frac{8}{1 + e^{-0.02t}} \to \frac{8}{1} = 8$.

3. (D) As $x \to 3^+$, $(\frac{1}{4})^{x^2} \to (\frac{1}{4})^{3/0^+} = (\frac{1}{4})^{\infty} = 0$.

4. (B) When the diver hits the water his position (height) is zero. Solve $s(t) = -8t^2 + 8t + 16 = 0$ for $t > 0$ to obtain $t = 2$. At this instant, the velocity is $s'(2)$ or $(-16t + 8)|_{t=2} = -24$ ft/s.

5. (A) With $x = t^2 - 4$ and $y = t/2$, the point $(12,2)$ corresponds to $t = 4$. The slope of the tangent line is $\frac{dy}{dx} = \frac{1/2}{-1/2} = -1$.

6. (B) With $H(x) = x^3 + g(f(x))$, we have $H'(-2) = 3(-2)^2 + g'(f(-2))f'(-2) = 12 + g'(5) \cdot 2 = 12 + 3(2) = 18$.

7. (A) As $\theta \to 0$, $\sin^{-1}(\theta) = \theta$ so $\sin^{-1}(\frac{2\theta}{\theta}) = \frac{8\theta}{2} \to \frac{8}{2} = 4$.

8. (D) With $f(x) = e^{\sin x}$, we have $f'(x) = e^{\sin x} \cos x$ and $f''(x) = e^{\sin x} \cos^2 x - e^{\sin x} \sin x$. Thus $f'''(\frac{\pi}{6}) = \sqrt{e}/4$.

9. (D) With $f(x) = (x^2 - 2x + 2)e^x$, we have $f(1) = e$ and $f'(1) = ((2x - 2)e^x + (x^2 - 2x + 2)e^x)|_{x=1} = e$. The tangent line is $y - e = e(x - 1)$ or $y = ex$.

10. (B) With $f(x) = \frac{1}{4}x^3 - x - 1$, we have $f(2) = 3$. Thus $g(3) = f^{-1}(3) = 2$. So $g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(3)}$. Since $f'(x) = \frac{3}{4}x^2 + 1$, we have $f''(2) = 4$ and $g'(3) = \frac{3}{4}$.

11. (C) At the intersection we have $r_1(t) = r_2(w)$ whence $1 - t = w - 2$ and $3 + t^2 = w^2$. So $w = 3 - t$. Hence $3 + t^2 = 9 - 6t + t^2$ or $6t = 6$. So $t = 1$ and $w = 2$.

12. (A) If $xy = 4$, then $\frac{dy}{dx}y + x \frac{dy}{dx} = 0$. When $x = 8$, $y = \frac{1}{2}$. Recall $\frac{dy}{dt} = 10$. Substituting, we have $5 + 8 \frac{dy}{dt} = 0$.

13. (E) For $f(x)$ is $\tan x$, we have $f'(x) = \sec^2 x$ and $f''(x) = 2\sec^2 x \tan x$. Thus $f'(\frac{\pi}{2}) = 1$, $f''(\frac{\pi}{2}) = 2$ and $f''(\frac{\pi}{2}) = 4$. The quadratic approximation of $f$ at $\frac{\pi}{2}$ is $Q(x) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2}) + \frac{1}{2}f''(\frac{\pi}{2})(x - \frac{\pi}{2})^2$ or $Q(x) = 1 + 2(x - \frac{\pi}{2}) + 2(x - \frac{\pi}{2})^2$.

14. (X) The problem is ill-posed. The point $(x,y) = (1,0)$ is not on the curve $(\sin \pi x + \cos \pi y)^3 = x^3y$, lest $1 = 0$ via substitution. All students were given full credit on this problem.

15. (A) With $f(x) = x^4$, we have $f'(x) = 4x^3$. Therefore, $f(2) = 16$ and $f'(2) = 32$. The linear approximation is $L(x) = f(2) + f'(2)(x - 2) = 16 + 32(x - 2)$. So $(201)^{3} = f(201) \approx L(201) = 16.32$.

16. (a) From the diagram, $x^2 + y^2 = 10^2$. Thus $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$ or $\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$. When $x = 6$, $y = \sqrt{100 - 36} = 8$. Recall $\frac{dx}{dt} = 4$. Hence $\frac{dy}{dt} = \frac{-6(4)}{8} = -3$ ft/s.

(b) Now $\sin \theta = \frac{y}{10}$, so $\cos \theta = \frac{dx}{dt} \frac{dt}{dt}$ or $\frac{dy}{dt} = -\frac{dx}{dt} \frac{dt}{dt}$.

17. $f(x) = \begin{cases} bx^2 + 3x + 2 & \text{if } x \leq -1; \\ ax + 5 & \text{if } x > -1. \end{cases}$

(a/b) As $f$ is piecewise polynomial, it is differentiable for $x \neq -1$. Make $f$ differentiable (and therefore also continuous) at $x = -1$.

- Equate left and right limits: $b - 1 = 5 - a$, whence $a = 6 - b$.
- Equate left and right slopes: $-2b = a$.
- Thus $6 - b = 3 - 2b$, so $b = 3, a = 9$.

So $f'(x) = \begin{cases} 3 - 6x & \text{if } x \leq -1 \\ 9 & \text{if } x > -1. \end{cases}$

Graph for #19 below!