MATH 151, SPRING 2015
COMMON EXAM III - VERSION A

LAST NAME: ___________________________________________ FIRST NAME: __________________________

INSTRUCTOR: __________________________
SECTION NUMBER: __________
UIN: __________________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: __________________________________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 3 points each

1. Below is the graph of $f'(x)$, the derivative of $f(x)$. At what value(s) of $x$ does $f(x)$ attain a local minimum?

(a) $x = -\frac{1}{2}$
(b) $x = 2$
(c) $x = -3$ and $x = 2$
(d) $x = -\frac{1}{2}$ and $x = 2$
(e) $x = -3$

2. \( \lim_{x \to 3^+} \ln \left( \frac{x - 3}{x} \right) = \)

(a) 1
(b) 0
(c) $\infty$
(d) $e$
(e) $-\infty$

3. Find all critical numbers for $f(x) = \sin^2 x + \cos x$, $0 < x < 2\pi$.

(a) $x = \pi$, $x = \frac{\pi}{3}$, $x = \frac{5\pi}{3}$

(b) $x = \frac{\pi}{3}$, $x = \frac{5\pi}{3}$

(c) $x = \pi$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$

(d) $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$

(e) $x = \pi$, $x = \frac{\pi}{6}$, $x = \frac{11\pi}{6}$
4. The number of bacteria in a culture is increasing according to the Law of Exponential Growth. If there are initially 125 bacteria in the culture and there are 350 bacteria after 2 hours, how many bacteria are present after 24 hours?

(a) $125 \left( \frac{5}{14} \right)^{12}$
(b) $125 \left( \frac{14}{5} \right)^{24}$
(c) $\frac{125}{2} \left( \frac{14}{5} \right)^{24}$
(d) $125 \left( \frac{14}{5} \right)^{12}$
(e) $\frac{125}{2} \left( \frac{5}{14} \right)^{12}$

5. What is the slope of the curve $f(x) = 3x \arcsin(x)$ at $x = -\frac{1}{2}$?

(a) $m = -\pi - \frac{3}{\sqrt{3}}$
(b) $m = -\frac{\pi}{2} - \frac{3}{\sqrt{3}}$
(c) $m = -\frac{\pi}{2} - \frac{3}{\sqrt{5}}$
(d) $m = \frac{7\pi}{2} - \frac{3}{\sqrt{3}}$
(e) $m = -\pi - \frac{3}{\sqrt{5}}$

6. Which of the following is equivalent to $2 \ln(x^2 + 3) - \ln(x) - \frac{1}{2} \ln(x^2 + 1)$?

(a) $\ln \left( \frac{(x^2 + 3)^2}{x\sqrt{x^2 + 1}} \right)$
(b) $\ln \left( \frac{(x^2 + 3)^2 \sqrt{x^2 + 1}}{x} \right)$
(c) $\ln \left( \frac{x(x^2 + 3)^2}{\sqrt{x^2 + 1}} \right)$
(d) $\ln \left( \frac{4(x^2 + 3)}{x(x^2 + 1)} \right)$
(e) $\frac{\ln(x^2 + 3)^2}{\ln x \sqrt{x^2 + 1}}$
7. \( \lim_{x \to 0} \frac{\arctan(2x)}{3x} = \)
   (a) 0
   (b) 1
   (c) \( \frac{2}{3} \)
   (d) 6
   (e) \( \frac{1}{3} \)

8. Find the absolute extrema for \( f(x) = 3x - x^3 \) on \([0, 3]\).
   (a) Absolute maximum is 2, Absolute minimum is -18
   (b) Absolute maximum is 0, Absolute minimum is -18
   (c) Absolute maximum is 2, Absolute minimum is -2
   (d) Absolute maximum is 2, Absolute minimum is 0
   (e) Absolute maximum is 4, Absolute minimum is -2

9. Find \( f'(x) \) for \( f(x) = x^{\ln x} \).
   (a) \( f'(x) = x^{\ln x} \left( \frac{2 \ln x}{x} \right) \)
   (b) \( f'(x) = x^{\ln x} \left( \frac{\ln x}{x} \right)^2 \)
   (c) \( f'(x) = x^{\ln x} \left( \frac{1}{x^2} \right) \)
   (d) \( f'(x) = \frac{2 \ln x}{x} \)
   (e) \( f'(x) = \ln x (x^{\ln x - 1}) \)
10. Which of the following is equivalent to sec(arctan $x$)?

(a) $\frac{1}{\sqrt{x^2 - 1}}$
(b) $\sqrt{x^2 + 1}$
(c) $\frac{1}{\sqrt{x^2 + 1}}$
(d) $\sqrt{x^2 - 1}$
(e) $\frac{x}{\sqrt{x^2 + 1}}$

11. $\arccos \left( \cos \left( \frac{4\pi}{3} \right) \right) =$

(a) $\frac{4\pi}{3}$
(b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$
(d) $-\frac{\pi}{3}$
(e) $-\frac{4\pi}{3}$

12. $\sum_{i=1}^{100} (2^i - 2^{i+1}) =$

(a) $1 - 2^{101}$
(b) $1 - 2^{100}$
(c) $2 - 2^{100}$
(d) $-2^{101}$
(e) $2 - 2^{101}$
13. Find the equation of the tangent line to the graph of \( f(x) = \ln(x^2) \) at \( x = e \).
   
   (a) \( y - 2e = 2e(x - e) \)

   (b) \( y - 2 = \frac{2}{e}(x - e) \)

   (c) \( y - 2e = \frac{2}{e}(x - e) \)

   (d) \( y - 2 = \frac{1}{e^2}(x - e) \)

   (e) \( y - \frac{2}{e} = 2e(x - e) \)

14. Solve for \( x \): \( \ln(x) + \ln(x + 1) = \ln(x + 4) \)
   
   (a) \( x = 2 \) and \( x = -2 \)

   (b) \( x = 3 \)

   (c) \( x = 3 \) and \( x = 2 \)

   (d) \( x = 4 \)

   (e) \( x = 2 \)

15. If \( f'(x) = (x - 2)^3(4 - x)^5(x + 1)^2 \), where does \( f(x) \) have a local minimum?
   
   (a) \( x = 4 \)

   (b) \( x = -1 \)

   (c) \( x = 4 \) and \( x = -1 \)

   (d) \( x = 2 \)

   (e) \( x = -1 \) and \( x = 2 \)
PART II: Work Out

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Consider \( f(x) = (x - 2)e^{3x} \).
   (i) (4 pts) Find the intervals where \( f(x) \) is increasing and decreasing.

   (ii) (2 pts) Find the local maximum and local minimum of \( f(x) \). If none, say *NONE*.

   (iii) (4 pts) Find the intervals where \( f(x) \) is concave up and concave down.

   (iv) (2 pts) Find the inflection point(s) of \( f(x) \). If none, say *NONE.*
17. (10 pts) A rectangle is bounded by the $x$-axis and the semicircle $f(x) = \sqrt{25-x^2}$ as shown below. What length and width should the rectangle have so that its area is a maximum?
18. (11 pts) Find \( \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^{3x} \).

19. (11 pts) Given that \( r''(t) = \langle e^t + t, \cos t - 1 \rangle \), \( r'(0) = \langle 1, -2 \rangle \) and \( r(0) = \langle 4, 12 \rangle \), find \( r(t) \).
20. Find the derivative of $f(x)$. Do not simplify.

(i) (4 pts) $f(x) = 4x \tan x$

(ii) (4 pts) $f(x) = \arccos(x^2)$

(iii) (3 pts) $f(x) = \log_3(\ln x)$