1. The curve \( y = \frac{x}{1 + x^2} \) is called a serpentine. Find an equation of the tangent line to the curve at \( x = 3 \).

2. Given these table values, compute \( p'(4) \) and \( q'(4) \) where \( p(x) = f(x)g(x) \) and \( q(x) = f(x)/g(x) \).

\[
\begin{array}{cccc}
  f(4) & g(4) & f'(4) & g'(4) \\
 2 & 5 & 6 & -3
\end{array}
\]
3. The table gives world population $P(t)$, in millions, where $t$ is in years and $t = 0$ corresponds to the year 1900. (a) Estimate the rate of population growth in 1920 by averaging the slopes of the two nearest secant lines. (b) Do the same for 2000.

<table>
<thead>
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<th>$t$</th>
<th>Pop.</th>
<th>$t$</th>
<th>Pop.</th>
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</tr>
<tr>
<td>50</td>
<td>2560</td>
<td>110</td>
<td>6870</td>
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</table>

4. The position of a particle is $s(t) = t^3 - 6t^2 + 9t$, $t \geq 0$, with $t$ in seconds and $s$ in meters. Answer these items.

(a) Draw a number line with $0 \leq t \leq 5$ at 1-second intervals. Show where velocity $v(t)$ is 0, +, −.

(b) Find the total distance traveled by the particle in the first 5 seconds. (Quick way: integrate speed.)
5. Find an equation of the tangent line to the curve 
\[ y = 2x \sin x \] at \((\frac{1}{2}\pi, \pi)\).

6. Graph \(f(x) = \frac{x}{\sqrt{1 - \cos 2x}}\) on calculator (or not).

(a) What type of discontinuity does \(f\) appear to have at 0? [No graph? Answer (b) first!]

(b) Analytically determine the type of discontinuity by computing left and right limits of \(f(x)\) at 0.
7. Given these table values, compute \( h'(1) \) and \( k'(1) \) where
\[
h(x) = f(g(x)) \text{ and } k(x) = g(f(x)).
\]

<table>
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<tr>
<th>( x )</th>
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<th>( g(x) )</th>
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<tr>
<td>3</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>9</td>
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</tbody>
</table>

8. Find all values of \( x \) at which \( f(x) = \sqrt{\frac{x^4 - x + 1}{x^4 + x + 1}} \) has horizontal tangents. (Use CAS and/or graph.)
9. There are two points on the slanted hyperbola
\[ x^2 - 4xy + y^2 = 4 \] corresponding to \( x = 0 \).

(a) First find the \( y \)-coordinates of these points.

(b) Compute slopes of tangent lines at the points.

10. Find equations of both tangent lines to the ellipse
\[ x^2 + 4y^2 = 36 \] that pass through the point \( P(12,3) \).
11. Given position $\mathbf{r}(t) = [-\frac{1}{2}t^2, t]$, $-4 \leq t \leq 4$, graph it. Compute velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$ at $t = 1$.

12. Let $\mathbf{r}(t) = [e^t \cos t, e^t \sin t]$. Compute velocity and speed at $t = 0$. 

[Diagram of Problem 11]
13. Obtain and identify a Cartesian equation for the parametric equations \( x = 3 \cos t, y = -2 \sin t \). Graph the curve, indicating direction of motion for increasing \( t \).

14. Given \( f(x) = xe^x \), find a formula for \( f^{(n)}(x) \), the \( n \)th derivative of \( f \). (Find some derivatives to see pattern.)
15. Find all points \((x, y)\) on the curve \(\mathbf{r}(t) = [e^{\cos t}, e^{\sin t}]\) where the tangent line is either horizontal or vertical.

16. Find the lowest point \((x, y)\) on the curve \(x = t^3 - 3t, y = t^2 + t + 1\). (The tangent line is horizontal there.)
17. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m$^3$/min. At what rate is the height of the water changing? Include appropriate units.

18. A ladder 5 m long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 m/s. What is the rate of change of the angle $\theta$ between the ladder and the ground when the ladder is 3 m from the wall?
19. The edge of a cube was found to be 30 cm with a possible error of \( \frac{1}{10} = 0.1 \) cm. Use differentials to estimate the maximum possible errors in the volume and surface area of the cube.

20. Say that \( g (2) = -4 \) and \( g' (x) = \sqrt{x^2 + 5} \) for all \( x \).

(a) Find the linear approximation \( L(x) \) of \( g \) at \( a = 2 \).

(b) Find the quadratic approximation \( Q(x) \) of \( g \) at \( a = 2 \).
21. Find the point \((x, y)\) where the curves \(y = x^3 - 3x + 4\) and \(y = 3(x^2 - x)\) are tangent to each other, that is, have a common tangent line. Give the line’s equation.

22. The figure below shows a circle of radius 1 inscribed in the parabola \(y = x^2\). Find its center \((h, k)\).
23. Find a formula for the inverse \( g(x) = f^{-1}(x) \) of the function \( f(x) = 2 + \sqrt{4 + 7x} \). Give the domain and range of \( g \).

24. Let \( g(x) = f^{-1}(x) \) be the inverse function of \( f(x) = x + e^x \). Compute \( g(1) \) and \( g'(1) \).