DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 4 points.

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
PART I: Multiple Choice. 4 points each

1. For which value of \( r \) does \( y = e^{rx} \) satisfy the equation \( y'' - 2y' + y = 0 \)?

   (a) 2
   (b) 1 \( key \ A \)
   (c) 0
   (d) -1
   (e) -2

2. \[ \lim_{x \to -\infty} \frac{2e^{-3x} - 3e^{3x}}{4e^{-3x} + 2e^{3x}} = \]

   (a) \( \infty \)
   (b) \( -\frac{3}{2} \)
   (c) \( -\frac{1}{6} \)
   (d) \( \frac{1}{2} \ \text{key} \ A \)
   (e) \( -\infty \)

3. A particle moves according to the equation of motion \( s(t) = t^2 - 2t + 3 \) where \( s(t) \) is measured in feet and \( t \) is measured in seconds. Find the total traveled distance in the first 3 seconds.

   (a) 2 feet
   (b) 3 feet
   (c) 5 feet \( key \ A \)
   (d) 6 feet
   (e) 11 feet
4. The vector function \( \mathbf{r}(t) = (t + e^{4t}, -t \cos(2t)) \), \( 0 \leq t \leq 2\pi \), represents the position of a particle at time \( t \). Find the acceleration vector of the object at \( t = \frac{\pi}{4} \).

(a) \( (1 + 4e^{\pi}, -1) \)
(b) \( (4e^{\pi}, \frac{\pi}{2}) \)
(c) \( (1 + 16e^{\pi}, \pi) \)
(d) \( (16e^{\pi}, 4) \) key A
(e) \( (16e^{\pi}, 4 + \pi) \)

5. Find \( \lim_{x \to 0} \frac{\sin^2(3x)}{x^2 \cos x} \).

(a) \( 3 \)
(b) \( 4 \)
(c) \( 6 \)
(d) \( 9 \) key A
(e) The limit does not exist.

6. Find the equation of the tangent line to the curve \( 2x^2y - 3y^2 = -11 \) at the point \( (2, -1) \).

(a) \( y = \frac{4}{7}x - \frac{1}{7} \)
(b) \( y = -\frac{4}{7}x + \frac{1}{7} \)
(c) \( y = -\frac{4}{7}x - \frac{1}{7} \)
(d) \( y = -\frac{4}{7}x + \frac{15}{7} \)
(e) \( y = \frac{1}{7}x - \frac{15}{7} \) key A
(7-8) Suppose $f$ and $g$ are differentiable functions which satisfy the following condition.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>3</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

7. Let $u(x) = f(x) \cdot g(x)$. Find $u'(1)$.
   
   (a) 0
   (b) -1
   (c) -3
   (d) -4 $key A$
   (e) It is impossible to determine the value from the information.

8. Let $v(x) = \frac{f(g(x))}{x^2}$. Find $v'(1)$.
   
   (a) 0
   (b) 2
   (c) 4
   (d) 6
   (e) 8 $key A$

9. Find the quadratic approximation for $f(x) = e^{x^2}$ at $x = 0$.
   
   (a) $1 + x^2$ $key A$
   (b) $1 + x + x^2$
   (c) $1 + 2x^2$
   (d) $1 + x + 2x^2$
   (e) 1
10. Find the value of $x$ where the tangent line to the graph of $f(x) = \frac{x}{\sqrt{x}}$ is parallel to the line $x - 4y = 10$.

(a) $\frac{1}{4}$  
(b) $\frac{1}{2}$  
(c) 1  
(d) 2  
(e) 4  key A

11. At what point does the curve parametrized by $x = t^2 - 6t + 5$, $y = t^2 + 4t + 3$ have a vertical tangent?

(a) $(-11, 35)$  
(b) $(-4, 24)$  key A  
(c) $(0, 8)$  
(d) $(5, 1)$  
(e) $(21, -1)$

12. Given the curve parametrized by $x = \sqrt{2t}$, $y = \sin \frac{\pi t}{2}$, find the slope of the line tangent to the curve at the point $(2, 0)$.

(a) $-\pi$  key A  
(b) $-\frac{1}{2}$  
(c) 2  
(d) $-\frac{1}{2}$  
(e) $\frac{1}{2}$
13. Find $f^{(2016)}(x)$ for $f(x) = \frac{-1}{(x+1)^2}$, $x \neq -1$.

(a) $f^{(2016)}(x) = -\frac{2016!}{(x+1)^{2017}}$

(b) $f^{(2016)}(x) = \frac{2016!}{(x+1)^{2017}}$

(c) $f^{(2016)}(x) = -\frac{2017!}{(x+1)^{2017}}$

(d) $f^{(2016)}(x) = \frac{2017!}{(x+1)^{2018}}$

(e) $f^{(2016)}(x) = -\frac{2017!}{(x+1)^{2018}}$ key A

14. Find $f'(x)$ if $f(x) = \sqrt{\cos(\sin x)}$.

(a) $\frac{-\sin(\sin x) \cdot \cos x}{2\sqrt{\cos(\sin x)}}$ key A

(b) $\frac{\cos x}{2\sqrt{\cos(\sin x)}}$

(c) $\frac{-\sin x \cdot \cos x}{2\sqrt{\cos(\sin x)}}$

(d) $\frac{-\sin^2 x \cdot \cos x}{2\sqrt{\cos(\sin x)}}$

(e) $\frac{\sin(\sin x) \cdot \cos x \cdot \sin x}{2\sqrt{\cos(\sin x)}}$

15. Suppose the linear approximation for the function $f(x)$ at $a = 3$ is given by $y = 2x - 2$. If $g(x) = \sqrt{f(x)}$, find the linear approximation for $g(x)$ at $a = 3$.

(a) $2 + \frac{1}{2}(x - 3)$ key A

(b) $2 + \sqrt{2}(x - 3)$

(c) $4 + 2(x - 3)$

(d) $4 + \frac{1}{2}(x - 3)$

(e) None of these
PART II: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (6 pts) Consider the parametric curve \( x = \frac{1}{3}t^3 - 2t^2 + 3t - 5 \) and \( y = t^2 + 2t \). Find the equation of the tangent line at \( t = 2 \).

key A: \( y = -6x - 18 \)

Point: \((x(2), y(2)) = \left(-\frac{11}{3}, 8\right)\)

Slope of tangent line: \( \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{\frac{2t + 2}{t^2 - 4t + 3}}{\frac{t^3 - 2t^2 + 3t - 5}{3}} \bigg|_{t=2} = -6 \)

\( \therefore \) The equation of the tangent line is \( y = -6x - 18 \).
17. (8 pts) Find \( f'(x) \). Do not simplify.

(a) (4 pts) \( f(x) = \frac{(4 - x)^2}{\tan x} \)

\[ f'(x) = \frac{-2(4 - x) \cdot \tan x - (4 - x)^2 \cdot \sec^2 x}{\tan^2 x} \]

(b) (4 pts) \( g(x) = \sin^4 \left( \pi^3 + \frac{1}{x^2} \right) \)

\[ g'(x) = 4 \sin^3 \left( \pi^3 + \frac{1}{x^2} \right) \cdot \cos \left( \pi^3 + \frac{1}{x^2} \right) \cdot \left( -\frac{2}{x^3} \right) \]
18. (8 pts) Let \( f(x) = \sqrt{x} \).

(a) (5 pts) Find the linear approximation for \( f(x) \) at \( x = 16 \).

\[ f(16) = 4, \quad f'(x) = \frac{1}{2\sqrt{x}}, \quad f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8} \]

\[ \therefore L(x) = 4 + \frac{1}{8}(x - 16) \]

(b) (3 pts) Use the linear approximation above to approximate \( \sqrt{16.03} \).

\[ \sqrt{16.03} = f(16.03) \approx L(16.03) = 4 + \frac{1}{8}(16.03 - 16) = 4 + \frac{3}{800} \]

\[ \text{key A: } 4 + \frac{1}{8}(0.03) \text{ or } 4 + \frac{3}{800} \text{ or } \frac{3203}{800} \]
19. (8 pts) Consider \( f(x) = \begin{cases} \frac{1}{2}x^2 + x + 1 & \text{if } x \leq 1 \\ bx - 1 & \text{if } x > 1 \end{cases} \)

(a) (6 pts) Find the value of \( a \) and \( b \) that make \( f(x) \) differentiable everywhere.

\text{key A: } a = 2, \ b = 5

- Continuous at \( x = 1 \).

\( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \)

\[ a(1)^2 + (1) + 1 = b(1) - 1 \]

\[ \Rightarrow \quad a + 2 = b - 1 \]

- Differentiable at \( x = 1 \).

\( \lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x) \)

\[ 2a(1) + 1 = b \]

\[ \Rightarrow \quad 2a + 1 = b \]

Then we have a linear system:

\[
\begin{align*}
    a + 2 & = b - 1 \\
    2a + 1 & = b
\end{align*}
\]

Therefore, \( a = 2, \ b = 5 \).

(b) (2 pts) For the value of \( a \) and \( b \) found above, find \( f'(x) \).

\text{key A: } f'(x) = \begin{cases} 4x + 1 & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}

\[ f'(x) = \begin{cases} 2ax + 1 & \text{if } x \leq 1 \\ b & \text{if } x > 1 \end{cases} \]

\[ \therefore \quad f'(x) = \begin{cases} 4x + 1 & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases} \]
20. (10 pts) Water is leaking out of an inverted conical tank at a rate of $1m^3/min$. The tank has height $6m$ and the diameter at the top is $4m$. At what rate is the water level changing when the height of the water is $3m$?

**key A**: $-\frac{1}{\pi}m/min$

Let $V$ is volume of water and $D$ is diameter of top of the tank. And $\frac{dV}{dt} = -1$ is given.

We use the equation of volume of conical tank; $V = \frac{1}{3}\pi r^2 h$ and since $r = \frac{1}{3}h$, it will be

$$V = \frac{\pi}{27}h^3$$

Take the derivative with respect to $t$, then

$$\frac{dV}{dt} = \frac{\pi}{9}h^2 \frac{dh}{dt}$$

$$\Rightarrow (-1) = \frac{\pi}{9}(3)^2 \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = -\frac{1}{\pi}m/min$$