# MATH 151, SPRING 2022
# COMMON EXAM I - VERSION A

LAST NAME(print): ______________________________ FIRST NAME(print): ______________________________

INSTRUCTOR: ______________________________

SECTION NUMBER: _______________

**DIRECTIONS:**

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

**"An Aggie does not lie, cheat or steal, or tolerate those who do."**

Signature: ______________________________________________

**PART I: Multiple Choice. 3 points each**

1. If $\mathbf{a} = \langle 1, 3\rangle$ and $\mathbf{b} = \langle 2, 2\rangle$, find $|3\mathbf{a} - 2\mathbf{b}|$

   (a) 1
   (b) 3
   (c) 4
   (d) $\sqrt{24}$
   (e) $\sqrt{26}$ ← correct

2. Given the points $A(1, -1)$ and $B(-3, 4)$, find a unit vector in the direction of the vector $\overrightarrow{AB}$.

   (a) $\left\langle \frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}} \right\rangle$
   (b) $\left\langle -\frac{4}{\sqrt{41}}, \frac{5}{\sqrt{41}} \right\rangle$ ← correct
   (c) $\left\langle -\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$
   (d) $\left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle$
   (e) None of these

3. The points $A(0, 1)$, $B(2, 0)$ and $C(3, -4)$ form a triangle. Find the angle $\angle BAC$.

   (a) $\angle BAC = \arccos\left(\frac{2}{\sqrt{85}}\right)$
   (b) $\angle BAC = \arccos\left(\frac{-6}{\sqrt{85}}\right)$
   (c) $\angle BAC = \arccos\left(\frac{-11}{\sqrt{170}}\right)$
   (d) $\angle BAC = \arccos\left(\frac{1}{\sqrt{170}}\right)$
   (e) $\angle BAC = \arccos\left(\frac{11}{\sqrt{170}}\right)$ ← correct

4. Find the work done by a force $\overrightarrow{F} = \langle 1, 3 \rangle$ moving an object from the point $P(1,1)$ to the point $Q(-1,2)$. (Force is measured in pounds and the distance is measured in feet)

   (a) $-1$ foot pounds
   (b) 0 foot pounds
   (c) 1 foot pounds ← correct
   (d) 4 foot pounds
   (e) 5 foot pounds

5. Find parametric equations for the line passing through the points $(1, -3)$ and $(-5, 8)$.

   (a) $x(t) = 1 + 11t,\ \ y(t) = -3 - 6t$
   (b) $x(t) = 1 - 6t,\ \ y(t) = -3 + 11t$ ← correct
   (c) $x(t) = 1 - 5t,\ \ y(t) = -3 + 8t$
   (d) $x(t) = 1 + 8t,\ \ y(t) = -3 - 5t$
   (e) $x(t) = 1 - 3t,\ \ y(t) = -5 + 8t$

6. Compute the scalar projection of the vector $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ onto the vector $\mathbf{a} = \mathbf{i} - \mathbf{j}$.

   (a) $\frac{1}{2}$
   (b) $\frac{\sqrt{2}}{2}$ ← correct
   (c) $\frac{\sqrt{5}}{5}$
   (d) $\frac{1}{5}$
   (e) $\frac{3\sqrt{2}}{2}$

7. The motion of a particle is given by the vector function $\mathbf{r}(t) = \langle 2 + \cos t, -1 + \sin t \rangle$. Which of the following describes the motion of the particle as $t$ increases?

(a) Clockwise around a circle
(b) Counterclockwise around an ellipse
(c) Counterclockwise around a circle $\leftarrow$ correct
(d) Clockwise around an ellipse
(e) None of these

8. Suppose $x$ and $y$ are real numbers and let $\mathbf{a} = \langle 3x, 7 \rangle$ and $\mathbf{b} = \langle -2, 5y \rangle$. If $\mathbf{a}$ is perpendicular to $\mathbf{b}$, what is the relationship between $x$ and $y$?

(a) $-6x + 35y = 1$
(b) $3x - 5y + 2 = 0$
(c) $-6x + 35y = 0$ $\leftarrow$ correct
(d) $-6x + 35y = 90$
(e) $3x + 5y + 11 = 0$

9. Simplify $\tan(\arccos(x))$ to an algebraic expression.

(a) $\dfrac{\sqrt{1-x^2}}{x}$ $\leftarrow$ correct
(b) $\dfrac{x}{\sqrt{1-x^2}}$
(c) $\dfrac{1}{x}$
(d) $\dfrac{x}{\sqrt{1+x^2}}$
(e) $\dfrac{1}{\sqrt{1+x^2}}$

10. Find the limit $\lim_{x \to -2^-} \frac{2x}{(x+2)(x-3)}$

(a) $2$

(b) $0$

(c) $-2$

(d) $-\infty$ $\leftarrow$ correct

(e) $\infty$

11. Evaluate $\lim_{x \to -3} \frac{x^2+3x}{x^2+x-6}$

(a) $\frac{3}{5}$ $\leftarrow$ correct

(b) $-\frac{3}{5}$

(c) $-\infty$

(d) $\infty$

(e) $0$

12. Evaluate $\lim_{t \to -\infty} \frac{3t-2}{\sqrt{4t^2+t+1}}$

(a) $\frac{3}{4}$

(b) $\frac{3}{2}$

(c) $0$

(d) $-\frac{3}{4}$

(e) $-\frac{3}{2}$ $\leftarrow$ correct

13. Evaluate $\lim_{x \to -\infty} \dfrac{7e^{-2x} - e^{4x}}{5e^{-2x} + 3e^{4x}}$

(a) $0$

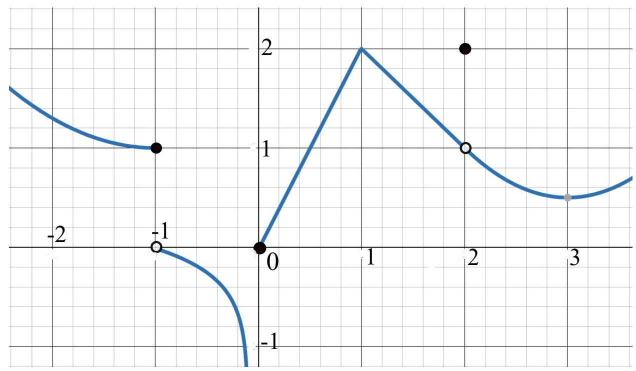
(b) $\dfrac{7}{5}$ ← correct

(c) $-\dfrac{1}{3}$

(d) $\infty$

(e) $-\infty$

Use the graph of $f$ to the right to answer Questions 14 and 15.




14. Which of the following statements is **false** concerning the limit of $f$?

(a) $\lim_{x \to -\mathbf{1}^-} f(x) = 1$

(b) $\lim_{x \to -\mathbf{1}^+} f(x) = 0$

(c) $\lim_{x \to \mathbf{0}} f(x) = 0$ ← correct

(d) $\lim_{x \to \mathbf{1}^-} f(x) = \lim_{x \to \mathbf{1}^+} f(x)$

(e) $\lim_{x \to \mathbf{2}} f(x) = 1$

15. Which of the following statements is **false** concerning the graph of $f$?

(a) $f$ is continuous from the left at $x = -1$.

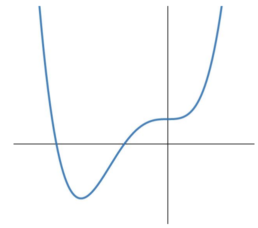
(b) $f$ has a jump discontiuity at $x = -1$.

(c) $f$ is continuous from the right at $x = 0$.

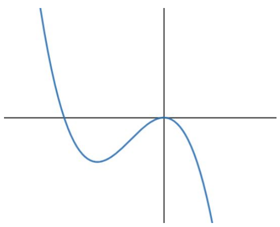
(d) $f$ is continuous and differentiable at $x = 1$ ← correct

(e) $f$ has a removable discontiuity at $x = 2$.

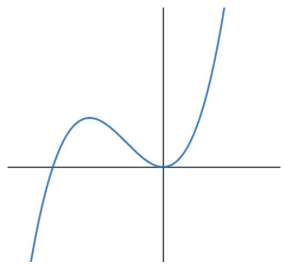
16. Consider the graph of $f(x)$ to the right.
Which of the following is the graph of the derivative, i.e., $f'(x)$?



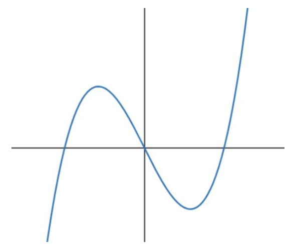
(a)



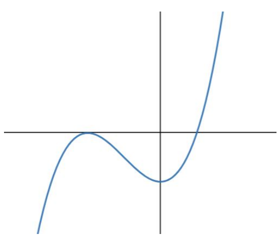
(b) ← correct



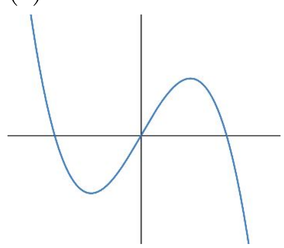
(c)



(d)



(e)



17. Evaluate limit $\lim_{x\to\infty} \left[\ln(x^2+3x+1) - \ln(x+x^3)\right]$.

(a) $-\infty$ ← correct
(b) $0$
(c) $\frac{2}{3}$
(d) $1$
(e) $\infty$

18. Find the limit $\lim_{x\to\infty} \arctan\left(\frac{1-x^3}{x^2-x}\right)$.

(a) $-\frac{\pi}{4}$
(b) $\frac{\pi}{4}$
(c) $-\frac{\pi}{2}$ $\leftarrow$ correct
(d) $\infty$
(e) $-\infty$

19. Find the horizontal and vertical asymptotes for $f(x) = \frac{(3-x)(2x+1)}{x^2-9}$

(a) $y = -3$, $x = -2$
(b) $y = -3$, $x = -3$, $x = 3$
(c) $y = -2$, $y = 2$, $x = -3$
(d) $y = -2$, $x = -3$ $\leftarrow$ correct
(e) $y = 2$, $x = -3$

20. Which of the following intervals contains a root to the equation $3x^3 - x^2 = 3$?

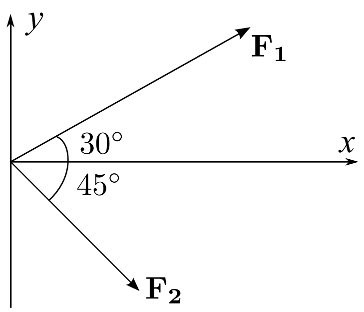
(a) $(-2, -1)$
(b) $(-1, 0)$
(c) $(0, 1)$
(d) $(1, 2)$ $\leftarrow$ correct
(e) $(2, 3)$

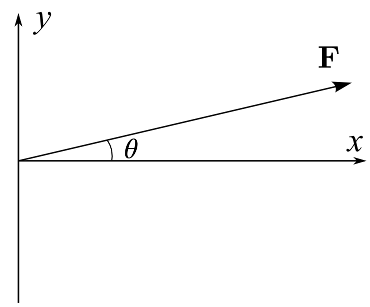
## PART II WORK OUT

**Directions**: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (9 points) Two forces act on an object as in the diagram below. $\mathbf{F_1}$ has a magnitude of 26 pounds and $\mathbf{F_2}$ has a magnitudes of 18 lbs.

    (a) Find the vectors $\mathbf{F_1}$, $\mathbf{F_2}$, and the resultant force $\mathbf{F}$. Your answers do not need to be simplified, but all trigonometric expressions which can be evaluated must be.




$$\mathbf{F_1} = |\mathbf{F_1}|\langle \cos 30°, \sin 30° \rangle = 26\langle \cos 30°, \sin 30° \rangle = 26 \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle = \langle 13\sqrt{3}, 13 \rangle$$

$$\mathbf{F_2} = |\mathbf{F_2}|\langle \cos(315°), \sin(315°) \rangle = 18 \langle \cos(45°), -\sin(45°) \rangle$$
$$= 18 \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle = \langle 9\sqrt{2}, -9\sqrt{2} \rangle$$

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \langle 13\sqrt{3}, 13 \rangle + \langle 9\sqrt{2}, -9\sqrt{2} \rangle = \langle 13\sqrt{3} + 9\sqrt{2}, 13 - 9\sqrt{2} \rangle$$

    (b) Find the resultant angle $\theta$ as shown in the diagram. Leave your answer in terms of an inverse trigonometric expression.

    Since $\tan\theta = \dfrac{13 - 9\sqrt{2}}{13\sqrt{3} + 9\sqrt{2}}$ we have

$$\theta = \tan^{-1}\left(\frac{13 - 9\sqrt{2}}{13\sqrt{3} + 9\sqrt{2}}\right).$$

22. (15 points) Evaluate these limits. Do not use the L'Hopital method.

(a) $\lim\limits_{x \to -3} \dfrac{x^2 + x - 6}{x^2 + 2x - 3}$

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 + 2x - 3} = \lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)(x-1)} = \lim_{x \to -3} \frac{x-2}{x-1} = \frac{-5}{-4} = \frac{5}{4}$$

(b) $\lim\limits_{x \to 2^-} \dfrac{3x^2 - 6x}{|x-2|}$

Since $x < 2$, $|x - 2| = -(x - 2)$. Thus we have

$$\lim_{x \to 2^-} \frac{3x^2 - 6x}{|x-2|} = \lim_{x \to 2^-} \frac{3x(x-2)}{-(x-2)} = \lim_{x \to 2^-} (-3x) = -6.$$

(c) $\lim\limits_{x \to -\infty} \left( \sqrt{x^2 + 5x + 3} + x \right)$

We may assume that $x < 0$. Using $x = -\sqrt{x^2}$, we have

$$\lim_{x \to -\infty} \left( \sqrt{x^2 + 5x + 3} + x \right) = \lim_{x \to -\infty} \frac{\left( \sqrt{x^2 + 5x + 3} + x \right) \left( \sqrt{x^2 + 5x + 3} - x \right)}{\sqrt{x^2 + 5x + 3} - x}$$

$$= \lim_{x \to -\infty} \frac{\left( x^2 + 5x + 3 \right) - x^2}{\sqrt{x^2 + 5x + 3} - x} = \lim_{x \to -\infty} \frac{5x + 3}{\sqrt{x^2 + 5x + 3} - x}$$

$$= \lim_{x \to -\infty} \frac{\dfrac{5x+3}{x}}{\dfrac{\sqrt{x^2 + 5x + 3} - x}{x}} = \lim_{x \to -\infty} \frac{5 + \dfrac{3}{x}}{\dfrac{\sqrt{x^2 + 5x + 3}}{-\sqrt{x^2}} - \dfrac{x}{x}}$$

$$= \lim_{x \to -\infty} \frac{5 + \dfrac{3}{x}}{-\sqrt{\dfrac{x^2 + 5x + 3}{x^2}} - 1} = \lim_{x \to -\infty} \frac{5 + \dfrac{3}{x}}{-\sqrt{1 + \dfrac{5}{x} + \dfrac{3}{x^2}} - 1}$$

$$= \frac{5}{-1 - 1} = -\frac{5}{2}$$

23. (7 points) Let $A$ and $B$ be constants. Consider the function

$$f(x) = \begin{cases} Ax^2 + Ax + 2, & \text{if } x < 2 \\ B, & \text{if } x = 2 \\ x^3 - x^2 + Ax, & \text{if } x > 2 \end{cases}$$

(a) Determine the value of $A$ for which $\lim_{x \to 2} f(x)$ exists.

If $\lim_{x\to 2} f(x)$ exists, two one sided limits $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$ must agree. We have

$$\lim_{x\to 2^-} Ax^2 + Ax + 2 = \lim_{x\to 2^+} x^3 - x^2 + Ax,$$

which implies $4A + 2A + 2 = 8 - 4 + 2A$ or $A = \frac{1}{2}$.

(b) Determine the value of $B$ for which $f(x)$ is continuous everywhere.

In order for $f(x)$ to be continuous, $\lim_{x\to 2} f(x) = f(2)$. With $A = \frac{1}{2}$ from (a) above, we have

$$\lim_{x\to 2^-} Ax^2 + Ax + 2 = 4A + 2A + 2 = 5 = B$$

Or,

$$\lim_{x\to 2^+} x^3 - x^2 + Ax = 8 - 4 + 2A = 5 = B$$

24. (9 points) Use the definition of the derivative to find $f'(x)$ for $f(x) = \dfrac{1}{2x+1}$. No points will be given for any shortcut formulas used.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{2(x+h)+1} - \dfrac{1}{2x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{2x+1-(2(x+h)+1)}{(2(x+h)+1)(2x+1)}}{h} = \lim_{h\to 0} \frac{\dfrac{-2h}{(2(x+h)+1)(2x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{-2h}{h\big(2(x+h)+1\big)(2x+1)} = \lim_{h\to 0} \frac{-2}{\big(2(x+h)+1\big)(2x+1)} = \frac{-2}{(2x+1)^2}$$

.

**DO NOT WRITE IN THIS TABLE.**

| Question | Points Awarded | Points |
|---|---|---|
| 1-20 | | 60 |
| 21 | | 9 |
| 22 | | 15 |
| 23 | | 7 |
| 24 | | 9 |
| | | 100 |