MATH 151, SPRING 2022

COMMON EXAM I - VERSION B

LAST NAME(print): ______________________________FIRST NAME(print): ______________________________

INSTRUCTOR: ______________________________

SECTION NUMBER: _______________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

PART I: Multiple Choice. 3 points each

1. Given the points $A(1,-1)$ and $B(-3,4)$, find a unit vector in the direction of the vector $\overrightarrow{AB}$.

   (a) $\left\langle -\frac{4}{\sqrt{41}}, \frac{5}{\sqrt{41}} \right\rangle$ $\leftarrow$ correct
   (b) $\left\langle \frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}} \right\rangle$
   (c) $\left\langle -\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$
   (d) $\left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle$
   (e) None of these

2. If $\mathbf{a} = \langle 1, 3 \rangle$ and $\mathbf{b} = \langle 2, 2 \rangle$, find $|3\mathbf{a} - 2\mathbf{b}|$

   (a) $\sqrt{26}$ $\leftarrow$ correct
   (b) $\sqrt{24}$
   (c) 4
   (d) 3
   (e) 1

3. Find the work done by a force $\mathbf{F} = \langle 1, 3 \rangle$ moving an object from the point $P(1,1)$ to the point $Q(-1,2)$. (Force is measured in pounds and the distance is measured in feet)

   (a) 5 foot pounds
   (b) 4 foot pounds
   (c) 1 foot pounds $\leftarrow$ correct
   (d) 0 foot pounds
   (e) -1 foot pounds

4. The points $A(0,1)$, $B(2,0)$ and $C(3,-4)$ form a triangle. Find the angle $\angle BAC$.

(a) $\angle BAC = \arccos\left(\dfrac{2}{\sqrt{85}}\right)$

(b) $\angle BAC = \arccos\left(\dfrac{-6}{\sqrt{85}}\right)$

(c) $\angle BAC = \arccos\left(\dfrac{11}{\sqrt{170}}\right)$ ← correct

(d) $\angle BAC = \arccos\left(\dfrac{-11}{\sqrt{170}}\right)$

(e) $\angle BAC = \arccos\left(\dfrac{1}{\sqrt{170}}\right)$

5. Compute the scalar projection of the vector $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ onto the vector $\mathbf{a} = \mathbf{i} - \mathbf{j}$.

(a) $\dfrac{1}{2}$

(b) $\dfrac{\sqrt{5}}{5}$

(c) $\dfrac{\sqrt{2}}{2}$ ← correct

(d) $\dfrac{1}{5}$

(e) $\dfrac{3\sqrt{2}}{2}$

6. Find parametric equations for the line passing through the points $(1,-3)$ and $(-5,8)$.

(a) $x(t) = 1 + 11t,\ \ y(t) = -3 - 6t$

(b) $x(t) = 1 - 5t,\ \ y(t) = -3 + 8t$

(c) $x(t) = 1 + 8t,\ \ y(t) = -3 - 5t$

(d) $x(t) = 1 - 6t,\ \ y(t) = -3 + 11t$ ← correct

(e) $x(t) = 1 - 3t,\ \ y(t) = -5 + 8t$

7. Simplify $\tan(\arccos(x))$ to an algebraic expression.

   (a) $\dfrac{x}{\sqrt{1-x^2}}$

   (b) $\dfrac{\sqrt{1-x^2}}{x}$ $\leftarrow$ correct

   (c) $\dfrac{1}{x}$

   (d) $\dfrac{x}{\sqrt{1+x^2}}$

   (e) $\dfrac{1}{\sqrt{1+x^2}}$

8. The motion of a particle is given by the vector function $\mathbf{r}(t) = \langle 2+\cos t, -1+\sin t\rangle$. Which of the following describes the motion of the particle as t increases?

   (a) Clockwise around a circle

   (b) Counterclockwise around a circle $\leftarrow$ correct

   (c) Counterclockwise around an ellipse

   (d) Clockwise around an ellipse

   (e) None of these

9. Suppose x and y are real numbers and let $\mathbf{a} = \langle 3x, 7\rangle$ and $\mathbf{b} = \langle -2, 5y\rangle$. If $\mathbf{a}$ is perpendicular to $\mathbf{b}$, what is the relationship between x and y?

   (a) $-6x+35y=1$

   (b) $3x-5y+2=0$

   (c) $-6x+35y=90$

   (d) $-6x+35y=0$ $\leftarrow$ correct

   (e) $3x+5y+11=0$

10. Evaluate $\lim_{x \to -3} \dfrac{x^2+3x}{x^2+x-6}$

(a) $-\infty$

(b) $-\dfrac{3}{5}$

(c) ∞

(d) 0

(e) $\dfrac{3}{5}$ $\leftarrow$ correct

11. Find the limit $\lim_{x \to -2^-} \dfrac{2x}{(x+2)(x-3)}$

(a) ∞

(b) 2

(c) 0

(d) -2

(e) $-\infty$ $\leftarrow$ correct

12. Evaluate $\lim_{x \to -\infty} \dfrac{7e^{-2x} - e^{4x}}{5e^{-2x} + 3e^{4x}}$

(a) 0

(b) $-\dfrac{1}{3}$

(c) ∞

(d) $-\infty$

(e) $\dfrac{7}{5}$ $\leftarrow$ correct

13. Evaluate $\displaystyle\lim_{t \to -\infty} \frac{3t-2}{\sqrt{4t^2+t+1}}$

(a) $\dfrac{3}{4}$

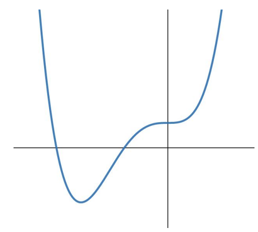
(b) $\dfrac{3}{2}$

(c) 0

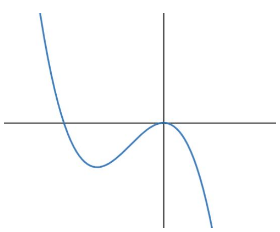
(d) $-\dfrac{3}{2}$ ← correct

(e) $-\dfrac{3}{4}$

14. Consider the graph of $f(x)$ to the right.
Which of the following is the graph of the derivative, i.e., $f'(x)$?



(a)



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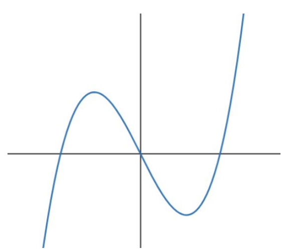
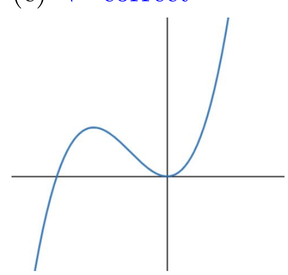


(c) ← correct



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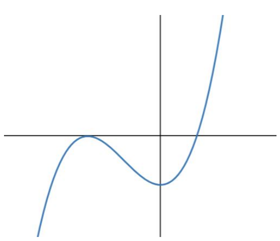

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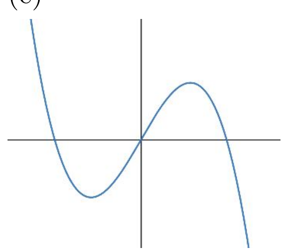
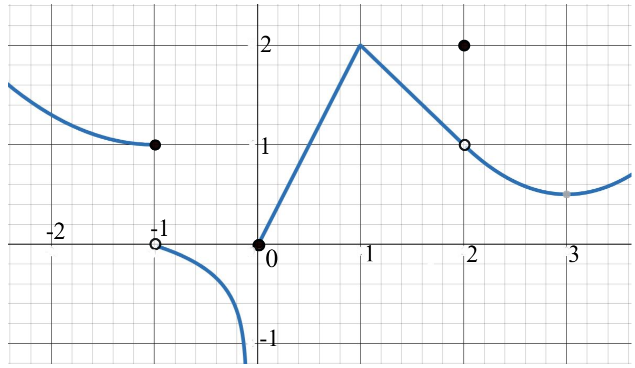


Use the graph of f to the right to answer Questions 15 and 16.



15. Which of the following statements is **false** concerning the limit of f?

(a) $\lim\limits_{x\to -\mathbf{1}^-} f(x) = 1$

(b) $\lim\limits_{x\to -\mathbf{1}^+} f(x) = 0$

(c) $\lim\limits_{x\to \mathbf{1}^-} f(x) = \lim\limits_{x\to \mathbf{1}^+} f(x)$

(d) $\lim\limits_{x\to \mathbf{0}} f(x) = 0$ $\leftarrow$ correct

(e) $\lim\limits_{x\to \mathbf{2}} f(x) = 1$

16. Which of the following statements is **false** concerning the graph of f?

(a) f is continuous from the left at $x = -1$.

(b) f has a jump discontiuity at $x = -1$.

(c) f is continuous and differentiable at $x = 1$ $\leftarrow$ correct

(d) f is continuous from the right at $x = 0$.

(e) f has a removable discontiuity at $x = 2$.

17. Which of the following intervals contains a root to the equation $3x^3 - x^2 = 3$?

(a) $(2, 3)$

(b) $(1, 2)$ $\leftarrow$ correct

(c) $(0, 1)$

(d) $(-1, 0)$

(e) $(-2, -1)$

18. Evaluate limit $\lim_{x\to\infty} \left[\ln(x^2+3x+1) - \ln(x+x^3)\right]$.

(a) 0

(b) $\frac{2}{3}$

(c) 1

(d) ∞

(e) $-\infty$ $\leftarrow$ correct

19. Find the limit $\lim_{x\to\infty} \arctan\left(\frac{1-x^3}{x^2-x}\right)$.

(a) $-\infty$

(b) $-\frac{\pi}{2}$ $\leftarrow$ correct

(c) $-\frac{\pi}{4}$

(d) $\frac{\pi}{4}$

(e) ∞

20. Find the horizontal and vertical asymptotes for $f(x) = \frac{(3-x)(2x+1)}{x^2-9}$

(a) $y = -3$, $x = -2$

(b) $y = -3$, $x = -3$, $x = 3$

(c) $y = -2$, $y = 2$, $x = -3$

(d) $y = 2$, $x = -3$

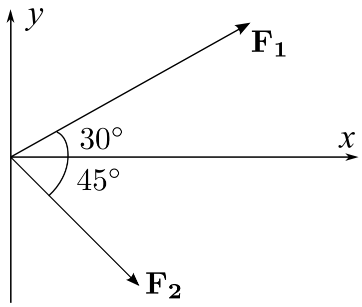
(e) $y = -2$, $x = -3$ $\leftarrow$ correct

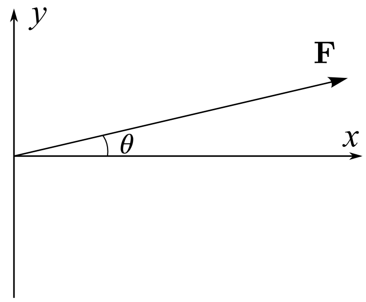
PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (9 points) Two forces act on an object as in the diagram below. $\mathbf{F_1}$ has a magnitude of 34 pounds and $\mathbf{F_2}$ has a magnitudes of 22 lbs.

(a) Find the vectors $\mathbf{F_1}$, $\mathbf{F_2}$, and the resultant force $\mathbf{F}$. Your answers do not need to be simplified, but all trigonometric expressions which can be evaluated must be.





$$\mathbf{F_1} = |\mathbf{F_1}|\langle \cos 30°, \sin 30° \rangle = 34\langle \cos 30°, \sin 30° \rangle = 34\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle = \langle 17\sqrt{3}, 17 \rangle$$

$$\mathbf{F_2} = |\mathbf{F_2}|\langle \cos(315°), \sin(315°) \rangle = 22\,\langle \cos(45°), -\sin(45°) \rangle$$
$$= 22\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle = \langle 11\sqrt{2}, -11\sqrt{2} \rangle$$

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \langle 17\sqrt{3}, 17 \rangle + \langle 11\sqrt{2}, -11\sqrt{2} \rangle = \langle 17\sqrt{3} + 11\sqrt{2}, 17 - 11\sqrt{2} \rangle$$

(b) Find the resultant angle θ as shown in the diagram. Leave your answer in terms of an inverse trigonometric expression.

Since $\tan\theta = \dfrac{17 - 11\sqrt{2}}{17\sqrt{3} + 11\sqrt{2}}$ we have

$$\theta = \tan^{-1}\left(\frac{17 - 11\sqrt{2}}{17\sqrt{3} + 11\sqrt{2}}\right).$$

22. (15 points) Evaluate these limits. Do not use the L'Hopital method.

(a) $\displaystyle\lim_{x\to-4} \frac{x^2+2x-8}{x^2+3x-4}$

$$\lim_{x\to-4} \frac{x^2+2x-8}{x^2+3x-4} = \lim_{x\to-4} \frac{(x+4)(x-2)}{(x+4)(x-1)} = \lim_{x\to-4} \frac{x-2}{x-1} = \frac{-6}{-5} = \frac{6}{5}$$

(b) $\displaystyle\lim_{x\to4^-} \frac{2x^2-8x}{|x-4|}$

Since $x<4$, $|x-4| = -(x-4)$. Thus we have

$$\lim_{x\to4^-} \frac{2x^2-8x}{|x-4|} = \lim_{x\to4^-} \frac{2x(x-4)}{-(x-4)} = \lim_{x\to4^-} (-2x) = -8.$$

(c) $\displaystyle\lim_{x\to-\infty} \left(\sqrt{x^2+3x+2}+x\right)$

We may assume that $x<0$. Using $x=-\sqrt{x^2}$, we have

$$\lim_{x\to-\infty} \left(\sqrt{x^2+3x+2}+x\right) = \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+3x+2}+x\right)\left(\sqrt{x^2+3x+2}-x\right)}{\sqrt{x^2+3x+2}-x}$$

$$= \lim_{x\to-\infty} \frac{\left(x^2+3x+2\right)-x^2}{\sqrt{x^2+3x+2}-x} = \lim_{x\to-\infty} \frac{3x+2}{\sqrt{x^2+3x+2}-x}$$

$$= \lim_{x\to-\infty} \frac{\frac{3x+2}{x}}{\frac{\sqrt{x^2+3x+2}-x}{x}} = \lim_{x\to-\infty} \frac{3+\frac{2}{x}}{\frac{\sqrt{x^2+3x+2}}{-\sqrt{x^2}}-\frac{x}{x}}$$

$$= \lim_{x\to-\infty} \frac{3+\frac{2}{x}}{-\sqrt{\frac{x^2+3x+2}{x^2}}-1} = \lim_{x\to-\infty} \frac{3+\frac{2}{x}}{-\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}-1}$$

$$= \frac{3}{-1-1} = -\frac{3}{2}$$

23. (7 points) Let A and B be constants. Consider the function

$$f(x) = \begin{cases} Ax^2 + Ax + 2, & \text{if } x < 2 \\ B, & \text{if } x = 2 \\ x^3 - 2x^2 - Ax, & \text{if } x > 2 \end{cases}$$

(a) Determine the value of A for which $\lim_{x \to 2} f(x)$ exists.

If $\lim_{x \to 2} f(x)$ exists, two one sided limits $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$ must agree. We have

$$\lim_{x \to 2^-} Ax^2 + Ax + 2 = \lim_{x \to 2^+} x^3 - 2x^2 - Ax,$$

which implies $4A + 2A + 2 = 8 - 8 - 2A$ or $A = -\frac{1}{4}$.

(b) Determine the value of B for which $f(x)$ is continuous everywhere.

In order for $f(x)$ to be continuous, $\lim_{x \to 2} f(x) = f(2)$. With $A = -\frac{1}{4}$ from (a) above, we have

$$\lim_{x \to 2^-} Ax^2 + Ax + 2 = 4A + 2A + 2 = \frac{1}{2} = B$$

Or,

$$\lim_{x \to 2^+} x^3 - 2x^2 - Ax = 8 - 8 - 2A = \frac{1}{2} = B$$

24. (9 points) Use the definition of the derivative to find $f'(x)$ for $f(x) = \dfrac{1}{3x+1}$. No points will be given for any shortcut formulas used.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{3(x+h)+1} - \dfrac{1}{3x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{3x+1-(3(x+h)+1)}{(3(x+h)+1)(3x+1)}}{h} = \lim_{h\to 0} \frac{\dfrac{-3h}{(3(x+h)+1)(3x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{-3h}{h\big(3(x+h)+1\big)(3x+1)} = \lim_{h\to 0} \frac{-3}{\big(3(x+h)+1\big)(3x+1)} = \frac{-3}{(3x+1)^2}$$

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DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-20		60
21		9
22		15
23		7
24		9
		100