Spring 1999
Math 151
Common Exam 3
Test Form A

PRINT: Last Name: ______________________________ **First Name:** ______________________

Signature: ____ **ID:** _________________________

Instructor's Name: ____ **Section #** _____________

INSTRUCTIONS

1. In **Part 1** (Problems 1–10), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.
2. In **Part 2** (Problems 11–16), write all solutions in the space provided. You may use the back of any page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS.

Part I	**Multiple Choice (5 points each)**	**No Calculators**

1. Consider $f(x) = 2x^3 - 9x^2 + 12x + 3$ on the interval $0 \le x \le 4$. The absolute maximum occurs at

   A. $x = 0$
   B. $x = 4$
   C. a point $x = c$ where $f'(c) = 0$
   D. a point $x = c$ where $f''(c) = 0$
   E. There is no absolute maximum.

2. $\sin^{-1}(\sin(3\pi/4)) =$

   A. $\frac{1}{2}\sqrt{2}$ B. $3\pi/4$ C. $-\frac{1}{2}\sqrt{2}$ D. $\dfrac{1}{\sin\left(\frac{1}{2}\sqrt{2}\right)}$ E. $\frac{\pi}{4}$

3. Find The derivative of $f(x) = \tan^{-1}(x^2 + 1)$.

   A. $f'(x) = \dfrac{2x}{x^2+1}$
   B. $f'(x) = -2x\csc^2(x^2+1)$
   C. $f'(x) = -\csc^2(x^2+1) + \tan^{-1}(2x)$
   D. $f'(x) = \dfrac{2x}{(x^2+1)^2+1}$
   E. $f'(x) = \dfrac{1}{(2x)^2+1}$

4. $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)\right) =$

   A. $\frac{1}{\sqrt{2}}$ B. $\frac{2}{\sqrt{5}}$ C. $\frac{1}{4}$ D. $\sqrt{\frac{2}{5}}$ E. $\frac{\pi}{2}$

5. $\lim_{x \to 0} \frac{\tan x - x}{x^3} =$

A. 0 B. $\frac{1}{3}$ C. does not exist D. $\frac{1}{2}$ E. 1

6. The inflection points of $f(x) = 3x^5 - 10x^4 + 7$ occur at:

A. $x = \frac{8}{3}$ B. $x = 0$ C. $x = 2$ D. $x = 0$ and $x = 2$ E. $x = 0$ and $x = \frac{8}{3}$

7. Consider the function defined by

$$f(x) = \begin{cases} x^2(x+2)^2, & x \leq 1, \\ 9 + x - x^2, & x > 1. \end{cases}$$

Find the x-values where the local maxima occur.

A. $x = 0$ and $x = -2$

B. $x = \frac{1}{2}$

C. $x = -1 \pm \frac{1}{3}\sqrt{3}$

D. $x = \pm 1$

E. $x = -1$

8. Let $f(x)$ be a continuous function on $0 \leq x \leq 4$. If $\int_0^2 f(x)dx = 3$, $\int_1^4 f(x)dx = 1$, and $\int_2^4 f(x)dx = 2$, then $\int_0^1 f(x)dx =$

A. 0 B. 1 C. 2 D. 3 E. 4

9. $\lim_{x\to 0} \tan^{-1}\left(\frac{1}{x^2}\right) =$

A. 0 B. $\frac{\pi}{2}$ C. $-\frac{\pi}{2}$ D. ∞ E. $-\infty$

10. Find an anti-derivative of $f(x) = \ln x - \frac{1}{x}$.

A. $x \ln x - x - \ln x$
B. $\frac{1}{x} + \frac{1}{x^2}$
C. $x \ln x - x + \frac{1}{x^2}$
D. $\frac{1}{x} + \ln x$
E. $\frac{1}{2}(\ln x)^2 - \ln x$

Part II	Partial Credit	Calculators Permitted

Calculators are permitted for *checking* answers but not for *supporting* them. Show your work to obtain credit. In particular, no credit will be given for derivatives found solely by formal differentiation on your calculators.

11. Find the dimensions of a right triangle which maximize the area with respect to the constraint that the sum of the hypotenuse and the base is equal to 1. (8 points)

12. Consider the function $f(x) = \frac{1}{x}$ on the interval $1 \leq x \leq 2$. Partition the interval into 5 equal sub-intervals, and calculate the Riemann sum associated with evaluating $f(x)$ at the mid-point of each sub-interval. (8 points)

13. A bacterial culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.

   (a) Find an expression for the number of bacteria after t hours. (4 points)

   (b) When will the population reach 30,000? (4 points)

14. Find the derivative of $f(x) = (x^2 + x + 1)^{\sin x}$. (8 points)

15. Let $f(x) = \dfrac{x}{(x+1)^2}$. Calculation shows that $f'(x) = \dfrac{1-x}{(1+x)^3}$ and $f''(x) = \dfrac{2(x-2)}{(x+1)^4}$.

(a) List the regions where $f(x)$ is increasing or decreasing. (5 points)

(b) List the regions where the curve $y = f(x)$ is concave up or concave down. (3 points)

(c) Find the horizontal and vertical asymptotes. (2 points)

16. Let $f(x)$ be a polynomial and suppose $x = 1$ and $x = 2$ are roots. By using a theorem, explain why the polynomial $f'(x)$ has a root that is strictly between 1 and 2. (8 points)