Fall 1998 Math 151 Common Exam 3

1. If $\tan\theta = 5/12$ then $\sec\theta = 13/12$.

2. $\pi = [y + 4\tan^{-1}(y)]' = y'[1 + 4(1+y^2)^{-1}] = 3y'$ so slope $= y' = \pi/3$.

3. $y = x^{\ln x}$ implies $\ln(y) = (\ln x)^2$. Differentiating both sides, $\frac{1}{y}\frac{dy}{dx} = 2(\ln x)\frac{1}{x}$.

4. Using L'Hospital's Rule, the limit is the same as $\lim\limits_{x\to 0} 2e^{2x}/1 = 2$.

5. Statements 1 and 4 are true.

6. The derivative is zero at $x = 2$. Checking the values of f at $x = 1, 2$ and 8, the smallest occurs at $x = 2$.

7. The function is increasing and concave down on (4,5) because

$$\begin{aligned} f'(x) &= 24x^2 - 4x^3 = 4x^2(6-x) > 0 \\ \text{and} \quad f''(x) &= 48x - 12x^2 = 12x(4-x) < 0 \end{aligned}$$

8. $f(x) = 2\sin x - 3\cos x + c$, $4 = f(0) = c - 3$ and $f(\pi/2) = 2 + c = 9$.

9. $\int_3^6 [2 + f(x)]dx = \int_3^6 2\ dx + \int_1^6 f(x)dx - \int_1^3 f(x)dx = 8$.

10. $A(x) = \int_0^x \sqrt{\sin t}\ dt$, $A'(x) = \sqrt{\sin x}$ and $A'(\pi/2) = 1$.

11. $\sum\limits_{i=2}^{5} (i-2)^2 = 0^2 + 1^2 + 2^2 + 3^2 = 14$.

12. $\int_1^2 (3x^2 - 2)dx = (x^3 - 2x)|_{x=1}^{x=2} = 5$.

13. $\int_0^{\ln 3} e^x\ dx = e^x|_{x=0}^{x=\ln 3} = e^{\ln 3} - e^0 = 3 - 1 = 2$.

14. (a) Let $P(t)$ be the population in thousands t years after year-end 1980. Since growth is exponential

$$P(t) = ce^{kt} \quad \text{and} \quad 154 = P(0) = c.$$

To solve for e^k,

$$261 = 154e^{15k} \Rightarrow e^k = \left(\frac{261}{154}\right)^{1/15}$$

and thus

$$P(t) = 154\left(\frac{261}{154}\right)^{t/15}$$

(b) The year-end population in 2000 is $P(20) \approx 311$ thousand.
(c) $400 = P(t) \Rightarrow$

$$(400/154) = (261/154)^{t/15} \Rightarrow$$

$t = 15\ln(400/154)/\ln(261/154) \approx 27.14$ years. Thus the population reaches 400 thousand about 27 years after the end of 1980.

15. If $y = [\cos x]^{1/x}$ then $\ln y = \frac{\ln(\cos x)}{x}$. Using L'Hospital's Rule,

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{-\sin x}{\cos x}/1 = -\tan 0 = 0$$

and so

$$\lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^0 = 1.$$

16. Let x be the side of the square base and h be the height. The cost in cents is

$$C = 4x^2 + 4xh.$$

Since the volume is $2000 = x^2 h$,

$$\begin{aligned} h &= 2000x^{-2}, \\ C &= 4x^2 + 8000x^{-1} \\ C' &= 8x - 8000x^{-2} = 8x^{-2}(x^3 - 1000). \end{aligned}$$

Use the first derivative test. C has a minimum when $C' = 0$, i.e., $x = 10$ cm; when $x = 10$, $h = 20$ cm.

17. Take antiderivatives twice. There are constants a and b so that

$$\begin{aligned} s'(t) &= a - 8.6t \\ s(t) &= b + at - 4.3t^2. \end{aligned}$$

Take $t = 0$ in the last two equations:

$$\begin{aligned} s'(0) &= a = \text{initial velocity} \\ s(0) &= b = 50. \end{aligned}$$

But you also know

$$0 = s(2) = 50 + 2a - 4(4.3),$$

so $a = -16.4$ m/s.

18. (a) The length of each interval is $\Delta x = 1$, so the left endpoint approximation is

$$\begin{aligned} L &= v(0) + v(1) + v(2) + v(3) \\ &= 0 + 30 + 45 + 52.5 \\ &= 127.5 \text{ feet} \end{aligned}$$

(b) The length of each interval is $\Delta x = 2$, so the midpoint approximation is

$$\begin{aligned} M &= v(1)2 + v(3)2 \\ &= (30)2 + (52.5)2 \\ &= 165 \text{ feet} \end{aligned}$$

(c) Yes, she fell more than 100 feet. The distance fallen is $s(4) - s(0) = \int_0^4 v(t)dt$. $v(t)$ is increasing so the integral is $\geq$ the left endpoint sum $= 127.5$.