MATH 151, FALL SEMESTER 2002

COMMON EXAMINATION I - VERSION A

Name (print):_____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–12), mark the correct choice on your ScanTron form (882-ES) using a No:2 pencil. *For your own record, also mark your choices on the exam.* ScanTrons will be collected from all examinees after one hour, and will *not* be returned.

2. Calculators may *not* be used in Part 1. The use of calculators is permitted *only after the first hour has elapsed and all ScanTrons have been collected.*

3. In Part 2 (Problems 13–17), present your solutions in the space provided. You may use the back of any page for rough work, but all work to be graded must be shown in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Part 1 – Multiple Choice (48 points)

Read each question carefully; each problem is worth **4 points**. *Calcluators are* **not** *allowed on this part of the exam.*

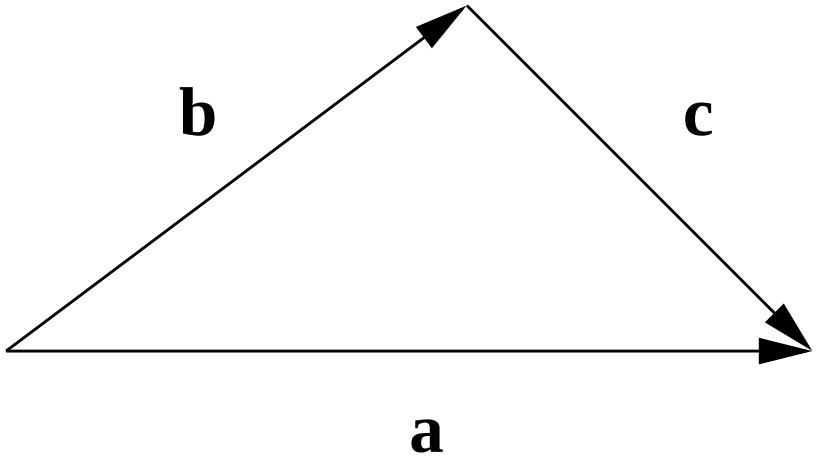
1. If $\mathbf{a} = \langle 2, 3\rangle$ and $\mathbf{b} = \langle 1, -2\rangle$, find $2\mathbf{a} - 3\mathbf{b}$.

(a) $\langle 1, 12\rangle$ (b) $\langle 1, 0\rangle$ (c) $\langle 4, 13\rangle$ (d) $\langle 8, 5\rangle$ (e) $\langle -1, -3\rangle$

2. Find the values of x for which the vectors $\langle 2, x\rangle$ and $\langle -3, 4\rangle$ are parallel.

(a) $3/2$ (b) $-8/3$ (c) $-3/2$ (d) $8/3$ (e) no such x exists

3. Consider and evaluate the following statements which pertain to the figure given below: (i) $\mathbf{a} + \mathbf{c} = \mathbf{b}$ (ii) $\mathbf{a} - \mathbf{b} = \mathbf{c}$ (iii) $\mathbf{a} + \mathbf{b} = \mathbf{c}$ (iv) $\|\mathbf{a}\| + \|\mathbf{c}\| = \|\mathbf{b}\|$, where $\| \ \|$ denotes the magnitude of a vector.



(a) only statement (i) is true (b) only statement (ii) is true (c) only statement (iii) is true (d) statements (i) and (iv) are both true (e) statements (ii) and (iv) are both true

4. Which of the following is a Cartesian equation for the parametric curve $\mathbf{r}(t) = \langle \cos t, 2\sin t\rangle$, $0 \le t \le 2\pi$?

(a) $x^2 + y^2 = 4$ (b) $x^2 + 4y^2 = 4$ (c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 1$ (e) $4x^2 - y^2 = 4$

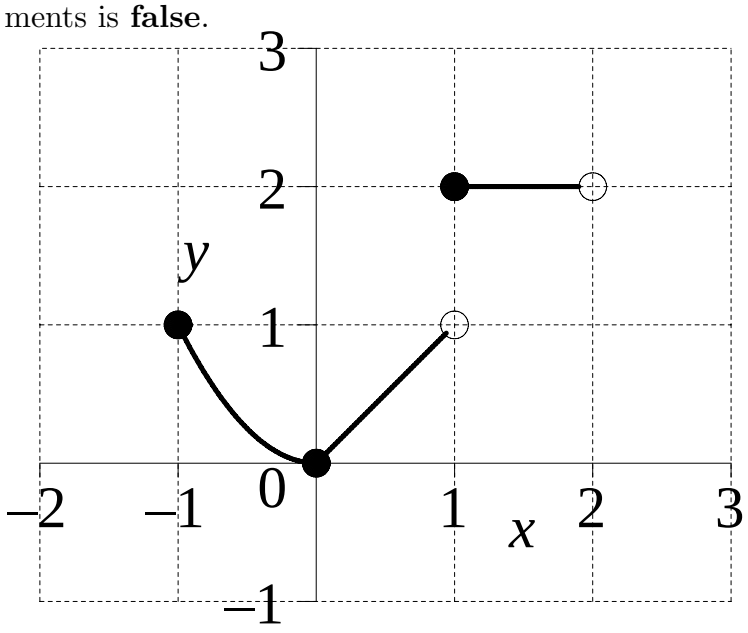
5. Find $\lim_{x\to 1^+} \dfrac{x-1}{|1-x|}$.

(a) -1 (b) 0 (c) 1 (d) does not exist (e) π^2

6. $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+1}+1}{x} =$

(a) 4 (b) 2 (c) 1 (d) 3 (e) $+\infty$

7. Refer to the graph of $y = f(x)$ given below, and decide which of the following statements is **false**.



(a) $\lim_{x\to 0} f(x) = 0$ (b) $\lim_{x\to 1^-} f(x) = 1$ (c) $\lim_{x\to 1^+} f(x) = 2$

(d) $\lim_{x\to 1} f(x) = 2$ (e) $\lim_{x\to 2^-} f(x) = 2$

8. Suppose f is a function and c is a fixed real number. The statement f *is continuous at* c is the same as saying that

(a) c belongs to the domain of f (b) f has a derivative at the point c

(c) Both $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$ exist (d) $\lim_{x\to c} f(x)$ exists

(e) $\lim_{x\to c} f(x) = f(c)$

9. Which of the following intervals is guaranteed to contain a solution of the equation $x^5 = 1 - 2x^3$?

(a) $[-2, -1]$ (b) $[-1, 0]$ (c) $[0, 1]$ (d) $[1, 2]$ (e) $[2, 3]$

10. If $f(x) = 1 + x\sqrt{x}$, find $f'(1)$.

(a) 5/2 (b) 7/5 (c) 3/2 (d) 2/5 (e) does not exist

11. If the tangent line to the graph of $y = f(x)$ at the point $(1, f(1))$ is perpendicular to the line $2x - 3y + 4 = 0$, what is $f'(1)$?

(a)2/3 (b) $-2/3$ (c) 3/2 (d) $-3/2$ (e) 4/3

12. If $f(2) = f'(2) = 3$, find $\lim_{x \to 2} \dfrac{(f(x))^2 - 9}{x - 2}$.

(a) 6 (b) 9 (c) 18 (d) does not exist (e) 0

Part II (55 points, including a bonus of 3)

Calculators are allowed for this part of the exam. Refer to the front page for further instructions.

13. Consider the triangle formed by the points $A(1,1)$, $B(3,4)$, and $C(2,-1)$.

(a) (`4 points`) Find the cosine of the angle θ included between the sides AB and AC.

(b) (`3 points`) Let D be the point on AC such that BD is perpendicular to AC. Determine the length of BD. (*Suggestion:* A quick way to do this is to use part (a).)

(c) (`3 points`) Obtain a vector equation of the straight line passing through B and C.

14. Suppose c is a fixed real number. Let

$$f(x) = \begin{cases} cx + 2, & \text{if } x < 2; \\ 0, & \text{if } x = 2; \\ c^2 + 2cx + 1, & \text{if } x > 2. \end{cases}$$

(a) (`2 points`) Find $\lim_{x \to 2^-} f(x)$.

(b) (`2 points`) Find $\lim_{x \to 2^+} f(x)$.

(c) (`4 points`) Find all possible values of c such that $\lim_{x \to 2} f(x)$ exists.

(d) (`2 points`) For each value of c found in part (c), evaluate the corresponding $\lim_{x \to 2} f(x)$.

(e) (`3 points - bonus`) Is there a value of c for which f is continuous at $x = 2$? Justify your answer.

15. Let $f(x) = \sqrt{5 + 2x}$.

(a) (`2 points`) What is the domain of f ?

(b) (`8 points`) Use the **definition of the derivative** to find $f'(-1)$. (**No** credit will be given for using any other method, correct answer notwithstanding.)

16. A particle moves according to a law of motion $s = \dfrac{\sqrt{t}}{t^2+1}$, where (the time) t is measured in seconds and (the displacement) s in feet.

(a) (`6 points`) Find the (instantaneous) velocity at time t, $t > 0$. Show all your steps.

(b) (`4 points`) Find the value(s) of t for which the instantaneous velocity is zero.

17. Suppose f and g are differentiable functions, and let $U(x) = \dfrac{1}{f(x)}$, $V(x) = g(x)f(x)$, and $S(x) = (f(x))^2$. Use the table of values given below to answer the questions that follow it.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	-1	3	2

(a) (`4 points`) Find $U'(1)$.

(b) (`4 points`) Find $V'(1)$.

(c) (`4 points`) Find $S'(1)$.