Spring 2007
Math 151
Common Exam 3
Test Form A

PRINT: Last Name _____ **First Name:** _____

Signature: _____ **ID:** _____

Instructor's Name: _____ **Section #** _____

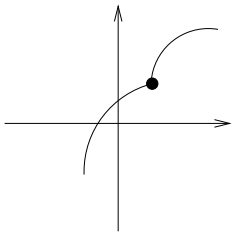
INSTRUCTIONS

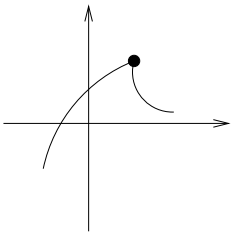
In **Part 1** (Problems 1–12), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.

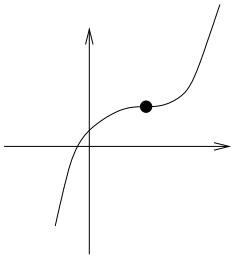
In **Part 2** (Problems 13–18), write all solutions in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS

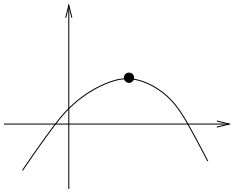
No Calculators Permitted

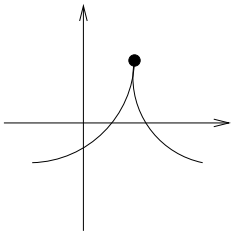
1. Which graph illustrates a local maximum which is also an inflection point?

(a) 

(b) 

(c) 

(d) 

(e) 

2. Consider the function $f(x) = 3x^4 - 8x^3 + 5$. Find the local minima of $f(x)$.

   (a) $x = 0$ and $x = 2$ (b) $x = 0$ and $x = \dfrac{4}{3}$

   (c) $x = 1$ (d) $x = 2$ (e) $x = \dfrac{4}{3}$

3. Find the derivative of $f(x) = \tan^{-1}(\ln x)$.

   (a) $f'(x) = \tan^{-1}\left(\dfrac{1}{x}\right)$ (b) $f'(x) = \left(\dfrac{1}{x}\right)\dfrac{1}{(\ln x)^2 + 1}$

   (c) $f'(x) = \dfrac{1}{x}\sec^{-2}(\ln x)$ (d) $f'(x) = \dfrac{x}{x^2 + 1}$

   (e) $f'(x) = \dfrac{1}{\tan^{-1} x}\left(\dfrac{1}{x^2 + 1}\right)$

4. Consider the graph of the function $f(x) = 10x^6 - 24x^5 + 15x^4$. Find the x-coordinates of the inflection points.

   (a) $x = 0$ (b) $x = 0$ and $x = 1$ (c) $x = 1$

   (d) $x = 1$ and $x = \dfrac{3}{5}$ (e) $x = 0, x = \dfrac{3}{5}$, and $x = 1$

5. $\displaystyle\sum_{i=1}^{50} \frac{3i - 7}{5} =$

   (a) 415 (b) -40 (c) $\dfrac{143}{5}$ (d) 23 (e) 695

6. Find the region where the function $f(x) = 2x^5 - 5x^4 - 10x^3$ is increasing.

   (a) $x > 3$ (b) $-1 < x < 3$ (c) $x < -1$ or $x > 3$

   (d) $x < -1$ or $0 < x < 3$ (e) $-1 < x < 0$ or $x > 3$

7. $\lim_{x\to 0} \dfrac{1-\cos^3 x}{\sin^2 x + \sin(2x)} =$

(a) does not exist (b) $-\dfrac{3}{4}$ (c) 0 (d) $\dfrac{3}{2}$ (e) 1

8. Find the derivative of $f(x) = x^{\ln x}$.

(a) $f'(x) = \dfrac{1}{x}x^{\ln x}$ (b) $f'(x) = 2\dfrac{\ln x}{x}x^{\ln x}$

(c) $f'(x) = x^{1/x}$ (d) $f'(x) = \dfrac{\ln x}{x}x^{\ln x}$

(e) $f'(x) = x^{2/x}$

9. $\cos\left(\sin^{-1}\left(-\dfrac{4}{5}\right)\right) =$

(a) $-\dfrac{3}{5}$ (b) $\dfrac{3}{5}$ (c) $-\dfrac{3}{4}$ (d) $\dfrac{3}{4}$ (e) does not exist

10. $e^{3\ln 2-1}\ln(5e^2) =$

(a) $\dfrac{8}{e}(\ln 5+2)$ (b) $\dfrac{e^3}{\ln 2}(\ln 5)^2$ (c) $e^3\ln 5$

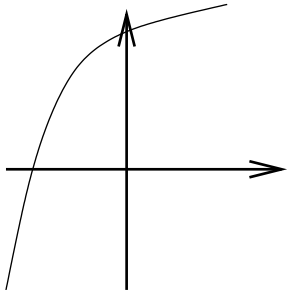
(d) $2\left(8+\dfrac{1}{e}\right)\ln 5$ (e) $\left(e^3\ln 2+\dfrac{1}{e}\right)(\ln 5+2e)$

11. Which function is an anti-derivative of $\dfrac{1}{\sqrt{1-x^2}}$?

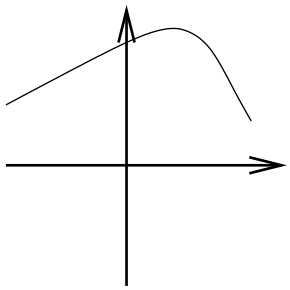
(a) $2\sqrt{1-x^2}$ (b) $\tan^{-1}x$ (c) $x(1-x^2)^{-3/2}$ (d) $\sin^{-1}x$ (e) $\dfrac{1}{x^2+1}$

12. Find the graph of the consistently increasing function whose derivative is consistently decreasing.

(a)



(b)



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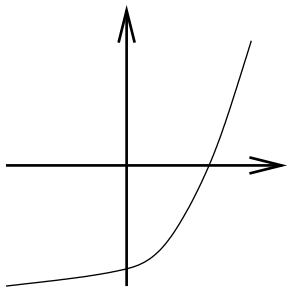
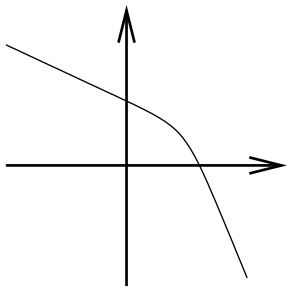
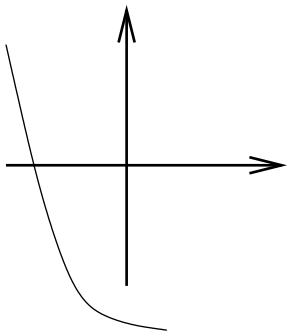


(d)



(e)



Show your work. No credit for unsupported answers will be given.

13. The radioactive decay law states that the rate of decrease in the amount of a radioactive isotope is always proportional to the amount remaining. The *half-life* of an isotope is the length of time over which the amount of the isotope is reduced by half through radioactive decay. Now suppose the initial amount of a given isotope is 7 grams and that 4 grams remain after $15 \ln 7 - 30 \ln 2$ years.

    (a) Find the formula for how the amount of isotope depends on time, (4 points)

    (b) Find the half-life of the isotope. (No decimal approximation is required. If you obtain an answer like $2^e e^2 \ln \pi$, then leave it alone. 3 points)

14. Calculate the following limits. (3 points each)

    (a) $\lim_{x \to 0} \dfrac{\tan^{-1} x}{\sin x}$

    (b) $\lim_{x \to \infty} \dfrac{\ln x}{\sqrt[3]{x}}$

(c) $\lim_{x \to 0} (1 + 2x)^{3/x}$

15. Consider the function $f(x) = \sqrt{x}$ and partition the interval $1 \leq x \leq 4$ into 6 equal sub-intervals. Calculate the Riemann sum for the points

$$x_1^* = 1, \quad x_2^* = \frac{16}{9}, \quad x_3^* = \frac{9}{4},$$
$$x_4^* = \frac{25}{9}, \quad x_5^* = \frac{49}{16}, \quad x_6^* = 4$$

of evaluation. (10 points)

16. Fencing is required to enclose a rectangular field with 7500 square feet of area. The north side requires higher quality fencing than the other sides. If the regular fencing costs $\$3\underline{00}$ per foot and the higher quality fencing costs $\$5\underline{00}$ per foot, how long must the north side be in order to minimize the cost? (10 points)

17. A car is accelerating along a straight highway, where the acceleration itself is increasing linearly—specifically, after t seconds, the acceleration is given by

$$a(t) = 30t + 8$$

in feet per second per second.

(a) Find the velocity $v(t)$ as a function of t if the initial velocity is $v(0) = 44$ ft./sec. (5 points)

(b) Find the position $x(t)$ as a function of t if the initial position is 100 feet down the road. (5 points)

18. Differentiate the function $f(x) = \dfrac{(2x-1)^3\sqrt{x+2}}{\sqrt[3]{5x-1}(3x+5)^2}$. (6 points) (Hint: The direct approach is unwise.)