Spring 2011
Math 152
Common Exam 3
Test Form A

PRINT: Last Name: _____ **First Name:** _____

Signature: _____ **ID:** _____

Instructor's Name: _____ **Section #** _____

INSTRUCTIONS

In **Part 1** (Problems 1–10), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.

In **Part 2** (Problems 11–15), write all solutions in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS.

No Calculators Permitted

(5 points each)

1. What is the power series representation of $f(x) = \ln(1+x)$ at $x = 0$?

(a) $\sum_{n=0}^{\infty} x^n$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$

(d) $\sum_{n=0}^{\infty} \left(\frac{x}{n+1}\right)^n$ (e) $\sum_{n=1}^{\infty} \ln(1+n)x^n$

2. Find the series which converges but does not converge absolutely.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$ (b) $\sum_{k=1}^{\infty} \frac{1}{k^2}$ (c) $\sum_{k=2}^{\infty} (-1)^k \left(1 - \frac{1}{k}\right)$

(d) $\sum_{k=0}^{\infty} (-1)^k e^{-k}$ (e) $\sum_{k=0}^{\infty} \left(-\frac{3}{2}\right)^k$

3. Which series converges absolutely?

(a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ (b) $\sum_{k=1}^{\infty} k(-1)^{k+1}$ (c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(d) $\sum_{k=1}^{\infty} \left(\sqrt{k+1} - \sqrt{k}\right)$ (e) $\sum_{k=1}^{\infty} \frac{(-1)^k}{3^k}$

4. Find the vector projection of $3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ onto $2\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$.

(a) -7 (b) $-\frac{7}{38}(2\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k})$ (c) $-\frac{7}{\sqrt{38}}$ (d) $-\frac{1}{2}(3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ (e) $-\frac{7}{\sqrt{14}}$

5. Find the unit vector in the direction of $3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k}$.

(a) $\frac{1}{169}(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$ (b) $\frac{1}{169}(-4\boldsymbol{i} + 12\boldsymbol{j} + 3\boldsymbol{k})$ (c) $13(-12\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$

(d) $\frac{1}{13}(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$ (e) $\frac{1}{5}(-3\boldsymbol{i} - 4\boldsymbol{j} + 12\boldsymbol{k})$

6. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{6^n}(2x-1)^n$.

(a) 3 (b) $\frac{1}{2}$ (c) 6 (d) $\frac{1}{3}$ (e) ∞

7. Find the equation of the sphere passing through (7,3,5) with center $(9, 0, -1)$.

(a) $(x-7)^2 + (y-3)^2 + (z-5)^2 = 82$

(b) $x + y + z = 15$

(c) $7(x-9) + 3y + 5(z+1) = 0$

(d) $(x-9)^2 + y^2 + (z+1)^2 = 83$

(e) $(x-9)^2 + y^2 + (z+1)^2 = 49$

8. Consider the 4th partial sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n2^n}$ as an approximation. Use the alternating series rule to obtain an upper bound on the absolute value of the error.

(a) $\frac{1}{64}$ (b) $\int_4^{\infty} \frac{dx}{x2^x}$ (c) $\frac{1}{384}$ (d) $\sum_{n=5}^{\infty} \frac{(-1)^n}{n2^n}$ (e) $\frac{1}{160}$

9. For which series is the ratio test inconclusive?

(a) $\sum_{k=1}^{\infty} \frac{k^3}{3^k}$ (b) $\sum_{k=1}^{\infty} \frac{2^k}{k^2}$ (c) $\sum_{k=1}^{\infty} \frac{3^{2k+1}}{2^{3k+5}}$ (d) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k!}$ (e) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{10}}$

10. If the force $\boldsymbol{F} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ is applied to a body that is constrained to move along the line segment from $A(1, -2, 0)$ to $B(3, -5, 2)$, which number is the cosine of the angle between $\boldsymbol{F}$ and this directed line segment?

(a) $\frac{13}{14}$ (b) $\frac{14}{\sqrt{13}\sqrt{17}}$ (c) $\frac{13}{\sqrt{14}\sqrt{17}}$ (d) $\frac{17}{\sqrt{13}\sqrt{14}}$ (e) $\frac{13}{17}$

Show your work. No credit will be given
to an unsupported answer. Partial credit is possible.

11. (5 points each) Find the Maclaurin series for each function.

(a) $f(x) = x^2 e^{2x}$

(b) $f(x) = \dfrac{\sin x}{x}$

(c) $f(x) = \dfrac{1}{1+4x^2}$

12. (10 points) Find the interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{1}{n3^{2n}}(x-1)^n.$$

13. Consider the function $f(x) = \sqrt{x}$.

(a) (6 points) Find the 2nd degree Taylor polynomial $T_2(x)$ at $x = \dfrac{9}{4}$.

(b) (6 points) Consider the interval $1 \leq x \leq 3$ $\left(\text{which contains } \dfrac{9}{4}\right)$ together with the Taylor inequality

$$|f(x) - T_n(x)| \leq \frac{M}{(n+1)!} \left|x - \frac{9}{4}\right|^{n+1}$$

over this interval, where

$$M = \max_{1 \leq x \leq 3} |f^{(n+1)}(x)|.$$

For $f(x) = \sqrt{x}$, use this inequality to find a number κ such that

$$|f(x) - T_2(x)| \leq \kappa, \qquad 1 \leq x \leq 3.$$

14. (6 points) Calculate the 3rd-degree Taylor polynomial of

$$f(x) = x^5 - 4x^4 + x^2 - 8x + 5$$

at $x = 1$.

15. (7 points) Express $\int_0^b e^{-x^2}\,dx$ as an infinite series by using the Maclaurin series of the integrand.