

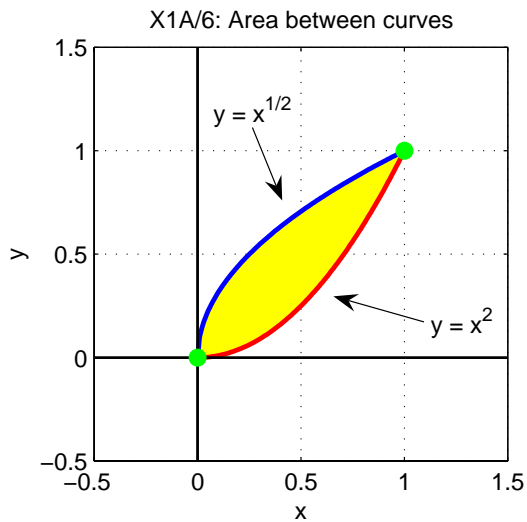
**Spring 2006 Math 152**  
**Exam 1A: Extras**  
**Mon, 20/Feb ©2006, Art Belmonte**

**Selected 2-D and 3-D Plots**

All plots were rendered with **MATLAB**.

**Problem 6**

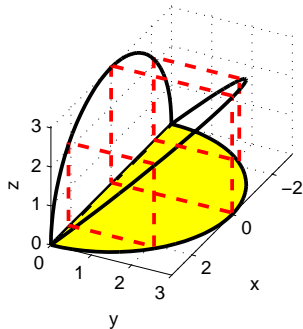
Here is a 2-D plot of the region between the curves.



**Problem 7**

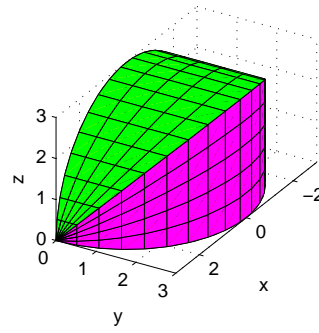
Here is a picture of the solid showing its semicircular base, its two upper edge boundaries, and a few square cross-sections.

X1A/7: Square cross-sectional slices



Here's another plot showing the surface patches (faces) of the solid. Clearly, this is NOT a solid of revolution!

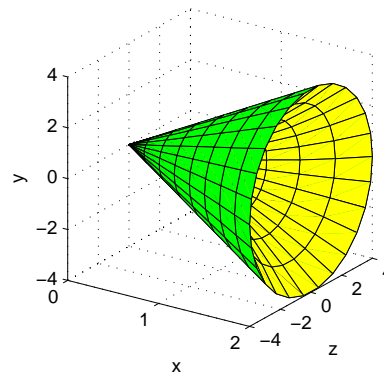
X1A/7: 3-D picture of solid; surface patches



**Problem 13**

Here is a 3-D plot of the solid of revolution.

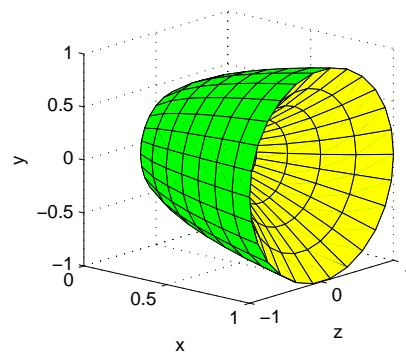
X1A/13: 3-D picture of solid



**Problem 14**

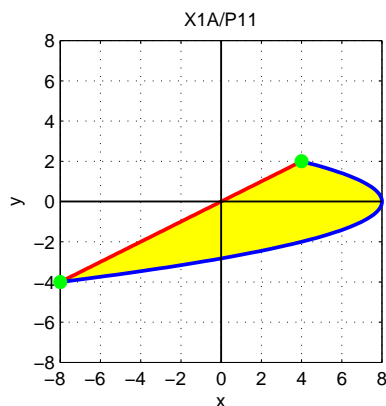
Here is a 3-D plot of the solid of revolution.

X1A/14: 3-D picture of solid



(Turn the page for alternative solutions to problems 11 and 12b.)

### Problem 11 by integrating with respect to $x$ .



Most students integrated with respect to  $y$ . One can integrate with respect to  $x$ , but must evaluate *two* integrals instead of one—and they are harder. With persistence, however, the result is the same.

$$\begin{aligned} V &= \int_{-8}^4 \frac{1}{2}x - (-\sqrt{8-x}) \, dx + \int_4^8 \sqrt{8-x} - (-\sqrt{8-x}) \, dx \\ &= \int_{-8}^4 \frac{1}{2}x + (8-x)^{1/2} \, dx + \int_4^8 2(8-x)^{1/2} \, dx \\ &= \left( \frac{1}{4}x^2 - \frac{2}{3}(8-x)^{3/2} \right) \Big|_{-8}^4 + \left( -\frac{4}{3}(8-x)^{3/2} \right) \Big|_4^8 \\ &= \left( 4 - \frac{16}{3} \right) - \left( 16 - \frac{128}{3} \right) + (0) - \left( -\frac{32}{3} \right) \\ &= -12 + 48 = 36 \end{aligned}$$

### Problem 12 via trig substitution

Most students used the Substitution Rule from Section 6.5, which is easier. It is possible, however, to use trig substitution. The form is  $a^2 - u^2$ . Choose  $x^5 = u = \sin \theta$ . Then  $5x^4 dx = \cos \theta d\theta$  or  $x^4 dx = \frac{1}{5} \cos \theta d\theta$ .

$$\begin{aligned} \int \frac{x^4}{\sqrt{1-x^{10}}} \, dx &= \int \frac{x^4}{\sqrt{1-(x^5)^2}} \, dx \\ &= \int \frac{\frac{1}{5} \cos \theta \, d\theta}{\cos \theta} = \int \frac{1}{5} \, d\theta \\ &= \frac{1}{5} \theta + C \\ &= \frac{1}{5} \sin^{-1}(x^5) + C \end{aligned}$$