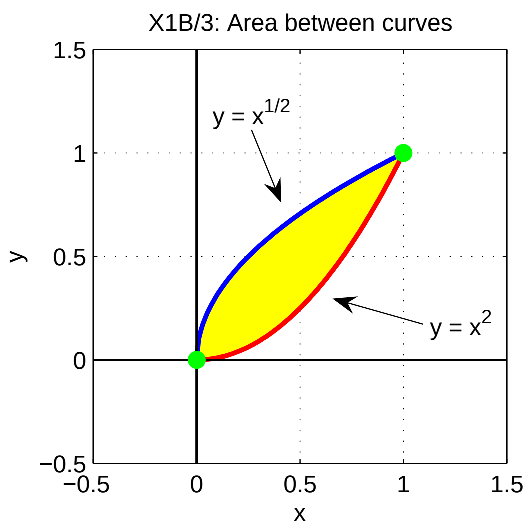
Spring 2006 Math 152
Exam 1B: Extras
Mon, 20/Feb

Selected 2_D and 3-D Plots

All plots were rendered with **MATLAB**.

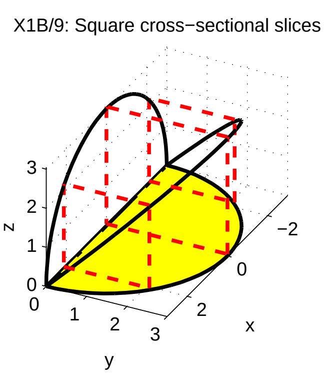
Problem 3

Here is a 2-D plot of the region between the curves.

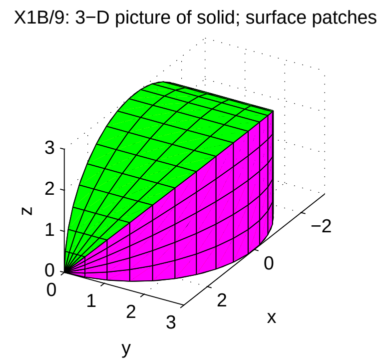


Problem 7

Here is a picture of the solid showing its semicircular base, its two upper edge boundaries, and a few square cross-sections.

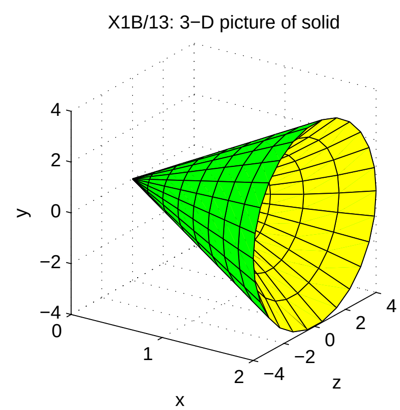


Here's another plot showing the surface patches (faces) of the solid. Clearly, this is NOT a solid of revolution!



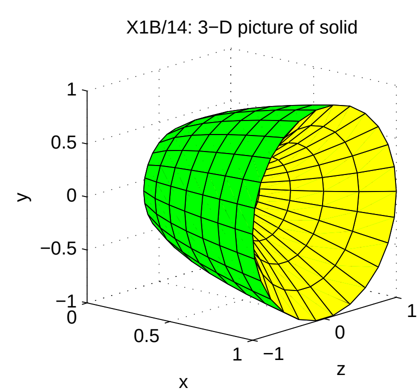
Problem 13

Here is a 3-D plot of the solid of revolution.



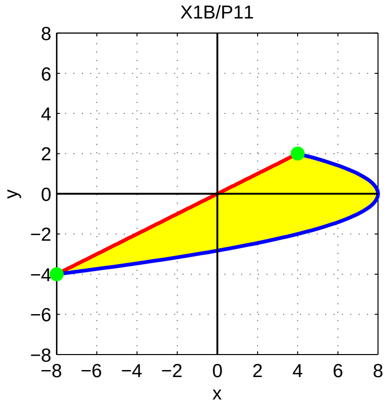
Problem 14

Here is a 3-D plot of the solid of revolution.



(Turn the page for alternative solutions to problems 11 and 12b.)

Problem 11 by integrating with respect to x.



Most students integrated with respect to y. One can integrate with respect to x, but must evaluate *two* integrals instead of one—and they are harder. With persistence, however, the result is the same.

$$
\begin{aligned}
V &= \int_{-8}^{4} \tfrac{1}{2}x - \left(-\sqrt{8-x}\right)\, dx + \int_{4}^{8} \sqrt{8-x} - \left(-\sqrt{8-x}\right)\, dx \\
&= \int_{-8}^{4} \tfrac{1}{2}x + (8-x)^{1/2}\, dx + \int_{4}^{8} 2\,(8-x)^{1/2}\, dx \\
&= \left(\tfrac{1}{4}x^2 - \tfrac{2}{3}(8-x)^{3/2}\right)\Big|_{-8}^{4} + \left(-\tfrac{4}{3}(8-x)^{3/2}\right)\Big|_{4}^{8} \\
&= \left(4 - \tfrac{16}{3}\right) - \left(16 - \tfrac{128}{3}\right) + (0) - \left(-\tfrac{32}{3}\right) \\
&= -12 + 48 = 36
\end{aligned}
$$

Problem 12 via trig substitution

Most students used the Substitution Rule from Section 6.5, which is easier. It is possible, however, to use trig substitution. The form is $a^2 - u^2$. Choose $x^5 = u = \sin\theta$. Then $5x^4 dx = \cos\theta\, d\theta$ or $x^4\, dx = \frac{1}{5}\cos\theta\, d\theta$.

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{1-x^{10}}}\, dx &= \int \frac{x^4}{\sqrt{1-\left(x^5\right)^2}}\, dx \\
&= \int \frac{\frac{1}{5}\cos\theta\, d\theta}{\cos\theta} = \int \frac{1}{5}\, d\theta \\
&= \frac{1}{5}\theta + C \\
&= \frac{1}{5}\sin^{-1}\left(x^5\right) + C
\end{aligned}
$$