Spring 2007 Math 152 Exam 1A Mon, 19/Feb/2007

SOLUTIONS: Executive Summary [method] ***Review on reverse*** →

1. (e) $\int_0^2 \frac{5x+7}{x^2+4x+3}\,dx = 4\ln 5 - 3\ln 3$ [partial fractions]
2. (a) $\int \pi r^2\,dy = \int \pi x^2\,dy = \pi\int_0^4 4-y\,dy = 8\pi$ [volume by cross-sections]
3. (b) $\int_0^2 x^3 e^{x^2}\,dx = \frac{3e^4+1}{2}$ [integration by parts]
4. (c) $f_{ave} = \frac{1}{3-0}\int_0^3 \frac{x}{\sqrt{x^2+16}}\,dx = \frac{1}{3}$ [substitution rule]
5. (d) $V = \int_0^{\sqrt{\pi}/2} 2\pi x\left(\cos\left(x^2\right) - \sin\left(x^2\right)\right)\,dx = \sqrt{2}\pi - \pi$ [volume by cylindrical shells]
6. (e) $V = \int y^2\,dx = \int_{-\pi/2}^{\pi/2} \cos x\,dx = 2$ [volume by cross-sections]
7. (c) $W_{total} = 20\times 10 + \int_0^{20} \frac{1}{2}x\,dx = 300$ [work lifting object and rope]
8. (d) $\int_1^e \frac{\ln x}{x^2}\,dx = 1 - \frac{2}{e}$ [integration by parts]
9. (b) $5 = F = kx = 2k$ implies $k = \frac{5}{2}$, whence $W = \int_0^4 \frac{5}{2}x\,dx = 20$. [Hooke's Law; work]
10. (d) $A = \int_{-2}^{3} (x+4) - \left(x^2-2\right)\,dx = \frac{125}{6}$ [area bewteen curves]
11. $\int \frac{x+1}{\left(x^2+4\right)^{3/2}}\,dx = \frac{x}{4\sqrt{x^2+4}} - \frac{1}{\sqrt{x^2+4}} + C$ [use trig sub $x = 2\tan\theta$]
12. $V = \int_0^2 \pi\,(8)^2 - \pi\left(8-x^3\right)^2\,dx = \frac{320\pi}{7}$ [volume by cross-sections]
13. $\int \cos^3 3\theta\,\sin^{-2} 3\theta\,d\theta = -\frac{1}{3}(\csc 3\theta + \sin 3\theta) + C$ [use basic trig identity]
14. $\int \cos t\cos 4t\,dt = \frac{1}{2}\left(\frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t\right) + C$ [use a trig product formula]
15. $W = \int_0^2 \frac{98}{10}(1000)\,\pi\,(4y)^{2/3}(2-y)\,dy = 35{,}280\pi$ [work done pumping water]

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Exam 1: Executive Summary
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Chapters and Sections

7: Applications of Integration

7.1	Areas Between Curves
7.2	Volume [by Cross-Sections]
7.3	Volumes by Cylindrical Shells
7.4	Work
7.5	Average Value of a Function

8: Techniques of Integration

8.1	Integration by Parts
8.2	Trigonometric Integrals
8.3	Trigonometric Substitution
8.4	Integration of Rational Functions by Partial Fractions

Fundamental Concepts (z = x or y)

7.1	$A = \int_a^b \lvert f(z) - g(z) \rvert \, dz$
7.2	$V = \int_a^b A(z) \, dz$
7.3	$V = \int 2\pi r L \, dz$
7.4	$W = \int_a^b F(z) \, dz$; Hooke's Law: $F(z) = kz$
7.5	$f_{ave} = \frac{1}{b-a} \int_a^b f(z) \, dz$; MVTI
8.1	$\int u \, dv = uv - \int v \, du$
8.2	Identities: basic; half, double; product
8.3	Trig with squares: $\sqrt{a^2 - u^2}$, $\sqrt{a^2 + u^2}$, $\sqrt{u^2 - a^2}$
8.4	PFD: Form; clear fracs; expand; collect; linear solve

Additional Details

- **MVTI**: f cont. on $[a, b] \implies \exists c \in [a, b]$ s.t. $f(c) = f_{ave}$
- **Basic**: $\sin^2 x + \cos^2 x = 1$ and derivations via division
- **Half**: $\cos^2 x, \sin^2 x = \frac{1}{2}(1 \pm \cos 2x)$, respectively
- **Double**: $\sin 2x = 2 \sin x \cos x$
- **Product**:

$$\begin{aligned} \sin A \cos B &= \tfrac{1}{2}\big(\sin(A-B) + \sin(A+B)\big) \\ \sin A \sin B &= \tfrac{1}{2}\big(\cos(A-B) - \cos(A+B)\big) \\ \cos A \cos B &= \tfrac{1}{2}\big(\cos(A-B) + \cos(A+B)\big) \end{aligned}$$

- **Trig subs**: $u = bx + c$; c (usually 0) and $a, b > 0$ constants

Expression	Substitution	Differential	Identity
$\sqrt{a^2 - u^2}$	$u = a \sin\theta$	$du = a \cos\theta \, d\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + u^2}$	$u = a \tan\theta$	$du = a \sec^2\theta \, d\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{u^2 - a^2}$	$u = a \sec\theta$	$du = a \sec\theta \tan\theta \, d\theta$	$\sec^2\theta - 1 = \tan^2\theta$

- **PFD** (Partial Fraction Decomposition): For proper rational expressions only (long divide beforehand if necessary).

Factor* in denominator	Terms in PFD summation
$(ax + b)^k$	$\frac{A}{ax+b}, \frac{B}{(ax+b)^2}, \cdots, \frac{C}{(ax+b)^k}$
$\left(ax^2 + bx + c\right)^k$	$\frac{Ax+B}{ax^2+bx+c}, \frac{Cx+D}{(ax^2+bx+c)^2}, \cdots, \frac{Ex+F}{(ax^2+bx+c)^k}$

*$ax^2 + bx + c$ is an irreducible quadratic; i.e., $b^2 - 4ac < 0$.

Notes

In water-pumping work, use the "march of the differentials." Here $\rho = 1000 \, \frac{\text{kg}}{\text{m}^3}$, $g = 9.8 \, \frac{\text{m}}{\text{s}^2}$, $\delta = 62.5 \, \frac{\text{lb}}{\text{ft}^3}$, $h(y)$ is the distance the layer is lifted, and $A(y)$ is the cross-sectional area of a layer. Also, $9.8 = \frac{98}{10} = \frac{49}{5}$ and $62.5 = \frac{125}{2}$, for help in hand work.

- **Metric**

$$\begin{aligned} dV &= A(y) \, dy \\ dm &= \rho \, dV = \rho A(y) \, dy \\ dF &= (dm) \, g = \rho g A(y) \, dy \\ dW &= (dF) \, D = \rho g A(y) \, h(y) \, dy \\ W &= \int dW = \int_c^d \rho g A(y) \, h(y) \, dy \end{aligned}$$

- **British**

$$\begin{aligned} dV &= A(y) \, dy \\ dF &= \delta \, dV = \delta A(y) \, dy \\ dW &= (dF) \, D = \delta A(y) \, h(y) \, dy \\ W &= \int dW = \int_c^d \delta A(y) \, h(y) \, dy \end{aligned}$$