Spring 2007 Math 152 Exam 3A Thu, 26/Apr/2007

SOLUTIONS: Executive Summary [method]

1. (c) The vector projection is $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|}\right)\dfrac{\mathbf{a}}{\|\mathbf{a}\|} = \dfrac{160}{129}\,\mathbf{i} - \dfrac{320}{129}\,\mathbf{j} - \dfrac{280}{129}\,\mathbf{k}$.

2. (b) The sum of the geometric series $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{5^n} = \sum_{n=1}^{\infty}\left(\frac{1}{5}\right)\left(-\frac{3}{5}\right)^{n-1}$ is $\dfrac{1/5}{1-\left(-\frac{3}{5}\right)} = \dfrac{1}{8}$.

3. (e) The cosine of the angle θ between the vectors $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is $\cos\theta = \dfrac{\mathbf{v}\cdot\mathbf{w}}{\|\mathbf{v}\|\,\|\mathbf{w}\|} = \dfrac{1}{14}$.

4. (d) The series $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$ is convergent, but not absolutely convergent by the Alternating Series Test and Integral Test.

5. (a) The series $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ converges by the Integral Test.

6. (c) The second-degree Taylor polynomial for $f(x) = \sqrt{x+3}$ about $a = 1$ is $T_2(x) = 2 + \frac{1}{4}(x-1) - \frac{1}{64}(x-1)^2$.

7. (b) The power series $\displaystyle\sum_{n=0}^{\infty}\frac{n^2}{2^n}(x-3)^n$ converges for $1 < x < 5$ via the Root Test (or Ratio Test) and Test for Divergence.

8. (d) The sequence $a_n = (-1)^n\dfrac{n}{n+2}$, $n \geq 1$, is bounded, yet nonmonotonic.

9. (a) The Maclaurin series expansion for the sine of t is $\sin t = \displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}t^{2n+1}$, valid for $t \in \mathbb{R}$.

   (b) Via algebra and term-by-term antidifferentiation, the power series expansion for the sine integral $\text{Si}(x) = \displaystyle\int_0^x \frac{\sin t}{t}\,dt$ is $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!\,(2n+1)}$, valid for $x \in \mathbb{R}$.

10. Consider the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-5)^{n+1}}{(2n+1)!}$.

    (a) It converges absolutely via the Ratio Test.

    (b) We have $s_2 = 3\frac{1}{8}$ or equivalent.

    (c) The Alternating Series Estimation Theorem yields an upper bound on the remainder in using s_2 to approximate the sum of the series, namely $125/1008$ or equivalent.

11. Via the Geometric Series Theorem, the power series expansion about $a = 0$ for $f(x) = \dfrac{1}{1+9x^2}$ is $\displaystyle\sum_{n=0}^{\infty}(-1)^n 9^n x^{2n}$.

    Its radius of convergence is $R = \frac{1}{3}$.

12. The function $f(x) = \dfrac{x}{1+x^3}$ has a Taylor series expansion about $a = 1$ given by $f(x) = \frac{1}{2} - \frac{1}{4}(x-1) - \frac{3}{8}(x-1)^2 + \cdots$, from which the relevant derivative and coefficient information may be ascertained.