MATH 152, FALL SEMESTER 2009
COMMON EXAMINATION II - VERSION A

Name (print):____________________________________

Signature: ____________________________________

Instructor's name: ____________________________________

Section No: ____________________________________

INSTRUCTIONS

1. In Part 1 (Problems 1–10), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well*. Graded ScanTrons will *not* be returned.

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 11–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Part 1 – Multiple Choice (50 points)

Each question is worth **5 points**. *Mark your responses on the ScanTron form and on the exam itself.*

1. The substitution $x = 2\sec\theta$ transforms the indefinite integral $\int \frac{\sqrt{x^2-4}}{x}\,dx$ into the following:

(a) $2\int \sec\theta\tan^2\theta\,d\theta$

(b) $3\int \tan^2\theta\,d\theta$

(c) $\int \tan^2\theta\,d\theta$

(d) $\int \sin\theta\,d\theta$

(e) $2\int \tan^2\theta\,d\theta$

2. Which of the following is the correct form of the partial-fraction decomposition for the rational function $\frac{x^3-2x+3}{(x-1)^3(x^2+5)^2}$?

(a) $\frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x-1} + \frac{D}{x^2+5} + \frac{E}{x^2+5}$

(b) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x^2+5} + \frac{E}{(x^2+5)^2}$

(c) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{(x^2+5)^2}$

(d) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{x^2+5}$

(e) $\frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x-1} + \frac{Dx+E}{x^2+5} + \frac{Fx+G}{x^2+5}$

3. Evaluate the improper integral $\int_0^\infty xe^{-2x}\,dx$.

(a) $-1/4$

(b) $1/9$

(c) $1/4$

(d) $-1/9$

(e) cannot be evaluated because the integral is divergent

4. Which of the following improper integrals are convergent?

$$\text{(I)} \quad \int_0^\infty \frac{e^{-x}}{x+1}\,dx \qquad \text{(II)} \quad \int_1^\infty \frac{x}{x^3+4}\,dx \qquad \text{(III)} \quad \int_1^\infty \frac{dx}{1+x}$$

(a) All three are convergent

(b) Only (II) and (III) are convergent

(c) Only (II) is convergent

(d) Only (I) and (II) are convergent

(e) All three are divergent

5. For which values of the fixed positive number p does the improper integral $\int_0^1 \frac{dx}{x^p}$ converge?

(a) The integral converges only for $0 < p < 1$

(b) The integral diverges for every $p > 0$

(c) The integral converges for every $p > 0$

(d) The integral converges only for $p > 1$

(e) The integral converges as long as $p \neq 1$

6. Compute the length of the curve $y = \frac{1}{3}(x^2+2)^{3/2}$, $0 \le x \le 1$.

(a) $8\pi/3$

(b) $14/3$

(c) $4/3$

(d) $28\pi/3$

(e) $3\pi/2$

7. Let C denote the parametric curve given by the equations $x(t) = 2\cos t$, $y(t) = 3\sin t$, $0 \le t \le \pi/2$. Which of the following represents the *area of the surface obtained by rotating C about the y-axis*?

(a) $4\pi \int_0^{\pi/2} \cos t\sqrt{9 - 5\sin^2 t}\, dt$

(b) $6\pi \int_0^{\pi/2} \cos t\sqrt{4 + 5\sin^2 t}\, dt$

(c) $6\pi \int_0^{\pi/2} \cos t\sqrt{9 - 5\sin^2 t}\, dt$

(d) $4\pi \int_0^{\pi/2} \cos t\sqrt{4 + 5\sin^2 t}\, dt$

(e) $\int_0^{\pi/2} \sqrt{9 - 5\sin^2 t}\, dt$

8. Given that $\lim_{n\to\infty} a_n = 2$ and $\lim_{n\to\infty} b_n = -3$, compute $\lim_{n\to\infty} \frac{a_n b_n}{2 + 3b_n^2}$.

(a) $3/7$

(b) $-3/7$

(c) $6/29$

(d) $-6/29$

(e) does not exist

9. Compute $\lim_{n\to\infty}[\ln(2n+1) - \ln(n)]$.

(a) $\ln(1/2)$

(b) $\ln(2)$

(c) 0

(d) ∞

(e) $-\infty$

10. Compute the sum of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$.

(a) 6

(b) 2/3

(c) 4

(d) 1

(e) 2

Part 2 (56 points)

Present your solutions to the following problems (11–16) in the space provided. **Show all your work** *neatly and concisely, and* **indicate your final answer clearly**. *You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

11. (`10 points`) Compute the following indefinite integral:

$$\int \frac{dx}{(x^2+4x+8)^2}$$

(Suggestion: Begin by completing squares.)

12. (`10 points`) Compute the following indefinite integral:

$$\int \frac{2x^2 + 2x + 1}{x^2(x^2 + 1)}\, dx$$

13. (`10 points`) Let C denote the circle of radius 1, centred at the origin. Let $0 < a < 1$ be a fixed number, and let Γ denote the portion of C which lies, *in the first quadrant*, between the y-axis and the line $x = a$. Let S denote the surface obtained by rotating Γ about the x-axis. Determine the value of a for which the area of S is $3/2$.

14. (i) (`5 points`) Compute

$$\lim_{n\to\infty} n^2 e^{-2n}.$$

Show all your steps clearly.

(ii) (`5 points`) Determine if the infinite series

$$\sum_{n=1} \frac{n}{2n+3}$$

converges or diverges. Explain your reasoning concisely but completely.

15. Decide whether each of the following infinite series is convergent or divergent. Explain your reasoning concisely but completely.

(i) (`5 points`)

$$\sum_{n=1}^{\infty} \frac{\sin^2(\sqrt{n})}{n(n+3)}$$

(ii) (`5 points`)

$$\sum_{n=1}^{\infty} \frac{1}{1+n^{1/3}}$$

16. (`6 points`) Suppose that f and g are functions which have continuous derivatives on the interval $[0, 1]$. Let C denote the curve given by the parametric equations $x(t) = f(t)$, $y(t) = g(t)$, $0 \leq t \leq 1$. Given that the length of C is L, compute the length of the curve given by the following set of parametric equations:

$$x(t) = g(1 - t), \quad y(t) = f(1 - t), \qquad 0 \leq t \leq 1.$$

Show all your steps clearly, and explain your reasoning fully.

QN	**PTS**
1–10	
11	
12	
13	
14	
15	
16	
TOTAL	