MATH 152, FALL SEMESTER 2009
COMMON EXAMINATION III - VERSION A

Name (print):_______________________________________

Signature: _______________________________________

Instructor's name: _______________________________________

Section No: _______________________________________

INSTRUCTIONS

1. In Part 1 (Problems 1–11), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well*. Graded ScanTrons will *not* be returned.

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 12–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Part 1 – Multiple Choice (55 points)

Each question is worth **5 points**. *Mark your responses on the ScanTron form and on the exam itself.*

1. If $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle 3, 2, 1 \rangle$, what is $2\mathbf{a} - 3\mathbf{b}$?

(a) $\langle -7, -2, -3 \rangle$

(b) $\langle -3, 2, 7 \rangle$

(c) $\langle 11, 10, 9 \rangle$

(d) $\langle 9, 10, 11 \rangle$

(e) $\langle -7, -2, 3 \rangle$

2. Find the cosine of the angle between the vectors $\mathbf{a} = \langle 1, 1, 2 \rangle$ and $\mathbf{b} = \langle -2, 3, 1 \rangle$.

(a) $3/\sqrt{84}$

(b) $1/\sqrt{84}$

(c) -1

(d) 1

(e) 0

3. Determine the value of the number x such that the vector $\langle x, 1, 2\rangle$ is orthogonal to the vector $\langle 3, 4, x\rangle$.

(a) $2/3$

(b) $4/5$

(c) $-2/3$

(d) $-4/5$

(e) no such x exists

4. Calculate the volume of the parallelepiped determined by the vectors $\langle 1, 0, 1\rangle$, $\langle 1, 2, 1\rangle$, and $\langle 0, 1, -1\rangle$.

(a) 3

(b) 4

(c) 5

(d) 1

(e) 2

5. Determine the radius of the sphere given by the equation $x^2 + y^2 + 2y + z^2 + z - 1 = 0$.

(a) $3/2$

(b) $9/4$

(c) 1

(d) $5/4$

(e) $\sqrt{5}/2$

6. Consider the pair of series

$$\text{(I)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}} \quad \text{and} \quad \text{(II)} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}.$$

Which of the following statements is true?

(a) Neither series is convergent.

(b) Neither series is absolutely convergent.

(c) (I) is absolutely convergent; (II) is convergent, but not absolutely convergent.

(d) (I) is convergent, but not absolutely convergent; (II) is absolutely convergent.

(e) Both series are absolutely convergent.

7. Which of the following is the Maclaurin expansion of the function $f(x) = e^{-x^2}$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-x^2)^n$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

8. Which of the following is the Maclaurin expansion of the function $f(x) = x^2 \sin(x)$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+3}}{(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+2}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+3)!} x^{2n+3}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+2)!} x^{2n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+3}$

9. Compute the sum of the infinite series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n)!}$.

(a) -1

(b) 1

(c) 0

(d) $-\pi$

(e) π

10. Suppose that the power series $\sum_{n=0}^{\infty} c_n(x-3)^n$ has radius of convergence 3. Consider the following pair of series:

$$\text{(I)} \quad \sum_{n=0}^{\infty} c_n 2^n \quad \text{and} \quad \text{(II)} \quad \sum_{n=0}^{\infty} c_n 4^n$$

Which of the following statements is true?

(a) Neither series is convergent.

(b) Both series are convergent.

(c) (I) is convergent, (II) is divergent.

(d) (I) is divergent, (II) is convergent.

(e) No conclusion can be drawn about either series.

11. Let (a_n) be a sequence of positive numbers, and suppose that the infinite series $\sum_{n=1}^{\infty} a_n$ is *convergent*. Which of the following statements regarding the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+a_n}$ is true?

(a) The series is convergent by the Ratio Test.

(b) The series is divergent by the Ratio Test.

(c) The series is convergent by the Comparion Test.

(d) The series is convergent by the Limit Comparion Test.

(e) The series is divergent by the Test for Divergence.

Part 2 (55 points)

Present your solutions to the following problems (12–16) in the space provided. **Show all your work** *neatly and concisely, and* **indicate your final answer clearly**. *You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.*

12. (i) (`7 points`) Find a vector perpendicular to the plane that passes through the points $P(1, 0, -1)$, $Q(0, 1, 1)$, and $R(1, 1, 1)$.

(ii) (`4 points`) Compute the area of the triangle with vertices P, Q, and R given above.

13. Consider the infinite series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}.$$

(i) (`5 points`) Determine whether the series is convergent. Explain your reasoning concisely but completely.

(ii) (`6 points`) Decide whether the series is absolutely convergent. Explain your reasoning concisely but completely.

14. Consider the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x-1)^n}{\sqrt{2n+1}}.$$

(i) (`5 points`) Compute the radius of convergence of the series. Explain your reasoning concisely but completely.

(ii) (`6 points`) Determine the interval of convergence of the series. Explain your reasoning concisely but completely.

15. (`11 points`) Consider the function

$$f(x) = x^3(x-1)^2, \qquad -\infty < x < \infty.$$

Compute T_3, the 3-rd degree Taylor polynomial of f at 1.

16. Consider the function

$$f(x) = \frac{1}{16 + x^4}.$$

(i) (`6 points`) Obtain the Maclaurin series representation for f.

(ii) (`5 points`) Use the series representation found in part (i) to express the definite integral $\int_0^1 f(x)\,dx$ as an infinite series.

QN	PTS
1–11	
12	
13	
14	
15	
16	
TOTAL	