MATH 152, Spring 2010
COMMON EXAM III - VERSION A

LAST NAME, First name (print): ______________________________

INSTRUCTOR: ______________________________

SECTION NUMBER: ____________

UIN: ______________________________

SEAT NUMBER: ______________________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam!*
3. In Part 2 (Problems 11-15), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________________

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-10		40
11		12
12		16
13		10
14		10
15		12
		100

PART I: Multiple Choice

1. (4 pts) What is the intersection of the sphere $(x+1)^2+(y-2)^2+(z-3)^2=25$ with the xz-plane?

   (a) $(x+1)^2+(z-3)^2=23$
   (b) $(x+1)^2+(y-2)^2=25$
   (c) The point $(0,2,0)$
   (d) The point $(-1,2,3)$
   (e) $(x+1)^2+(z-3)^2=21$

2. (4 pts) The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x+1)^n(2n+1)!}{10^n n!}$ is:

   (a) $I=\{-1\}$
   (b) $I=(-11,9)$
   (c) $I=(-\infty,\infty)$
   (d) $I=[-11,9)$
   (e) $I=\left(-\frac{1}{10},\frac{1}{10}\right)$

3. (4 pts) Which of the following series converge absolutely?

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
   (b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$
   (c) $\sum_{n=1}^{\infty} (-1)^n$
   (d) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$
   (e) All of the above series are absolutely convergent.

4. (4 pts) $\arctan(x^3) =$

(a) $\sum_{n=0}^{\infty} \frac{x^{6n+3}}{2n+1}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{6n+1}$

(e) $\sum_{n=0}^{\infty} \frac{x^{6n+1}}{6n+1}$

5. (4 pts) Find the unit vector in the direction of $\mathbf{b} - \mathbf{a}$ where $\mathbf{a} = \langle 0, 2, 1 \rangle$ and $\mathbf{b} = \langle 1, 1, 3 \rangle$.

(a) $\left\langle \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$

(b) $\langle 1, -1, 2 \rangle$

(c) $\langle -1, 1, -2 \rangle$

(d) $\left\langle -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}} \right\rangle$

(e) $\left\langle \frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle$

6. (4 pts) Given the triangle with vertices $A(2, -2, 5)$, $B(1, 1, 4)$ and $C(3, 1, 3)$, find the cosine of the angle at B.

(a) $\frac{1}{\sqrt{55}}$

(b) $\frac{3}{\sqrt{55}}$

(c) $\frac{1}{\sqrt{11}}$

(d) $\frac{3}{\sqrt{11}}$

(e) None of the above.

7. (4 pts) Which of the following is equal to $\frac{e^x - 1 - x}{x^2}$?

(a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{x^n}{(n+2)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+2)!}$

8. (4 pts) If we represent $\dfrac{1}{9+4x^2}$ as a power series centered at zero, what is the associated radius of convergence?

(a) $R = \dfrac{9}{4}$

(b) $R = \dfrac{3}{2}$

(c) $R = \dfrac{2}{3}$

(d) $R = \dfrac{4}{9}$

(e) $R = \dfrac{1}{2}$

9. (4 pts) Using the Alternating Series Estimation Theorem, how many terms of the series do we need to add in order to find the sum of the series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^2}$ with error less than $\dfrac{1}{60}$?

(a) $n = 6$

(b) $n = 5$

(c) $n = 4$

(d) $n = 7$

(e) $n = 3$

10. (4 pts) The series $\sum_{n=0}^{\infty} \dfrac{(-1)^n n^2}{n^2+1}$

(a) diverges by the Alternating Series Test.

(b) converges, but not absolutely.

(c) converges absolutely.

(d) diverges by the Test for Divergence.

(e) none of the above.

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (12 pts) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-3)^n(2x-1)^n}{n}$.

12. (i) (4 pts) Find a Maclaurin Series representaton for $f(x) = \sin\left(\frac{x^2}{3}\right)$.

(ii) (6 pts) Using the result in part (i), write $\int_0^1 \sin\left(\frac{x^2}{3}\right)\, dx$ as an infinite series.

(iii) (6 pts) Using the series found in part (ii), find the sum of the first three nonzero terms to estimate $\int_0^1 \sin\left(\frac{x^2}{3}\right)\, dx$. Give an upper bound on the error.

13. (10 pts) Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^4}$ converge absolutely? Justify your answer.

14. (10 pts) Find the Taylor series for $f(x) = \ln x$ centered at $a = 4$. Do not examine convergence.

15. Let $f(x) = e^{2-x}$.

(i) (6 pts) Give the fourth degree Taylor polynomial for $f(x)$ centered at $a = 2$.

(ii) (6 pts) Use Taylor's Inequality to give a bound on the error when using the polynomial from (i) to estimate $f(x)$ on the interval $[-1, 5]$.

Taylor's Inequality: $|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$, where $|f^{(n+1)}(x)| \leq M$ for x in an interval containing a.