

**MATH 152, Spring 2010  
COMMON EXAM III - VERSION B**

LAST NAME, First name (print): \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

UIN: \_\_\_\_\_

SEAT NUMBER: \_\_\_\_\_

**DIRECTIONS:**

1. The use of a calculator, laptop or computer is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam!*
3. In Part 2 (Problems 11-15), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form*.

THE AGGIE CODE OF HONOR

**“An Aggie does not lie, cheat or steal, or tolerate those who do.”**

Signature: \_\_\_\_\_

**DO NOT WRITE BELOW!**

| Question | Points Awarded | Points |
|----------|----------------|--------|
| 1-10     |                | 40     |
| 11       |                | 12     |
| 12       |                | 16     |
| 13       |                | 10     |
| 14       |                | 10     |
| 15       |                | 12     |
|          |                | 100    |

PART I: Multiple Choice

1. (4 pts) The series  $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{n^2 + 1}$

- (a) Diverges by the Test for Divergence
- (b) Converges, but not absolutely
- (c) Diverges by the Alternating Series Test
- (d) Converges absolutely
- (e) None of the above

2. (4 pts) Which of the following is equal to  $\frac{e^x - 1 - x}{x^2}$ ?

- (a)  $\sum_{n=0}^{\infty} \frac{x^n}{(n+2)!}$
- (b)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$
- (c)  $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$
- (d)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+2)!}$
- (e)  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

3. (4 pts) Find a unit vector in the direction of  $\mathbf{b} - \mathbf{a}$  where  $\mathbf{a} = \langle 0, 2, 1 \rangle$  and  $\mathbf{b} = \langle 1, 1, 3 \rangle$ .

- (a)  $\langle 1, -1, 2 \rangle$
- (b)  $\left\langle \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$
- (c)  $\left\langle -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}} \right\rangle$
- (d)  $\left\langle \frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle$
- (e)  $\langle -1, 1, -2 \rangle$

4. (4 pts) Using the Alternating Series Estimation Theorem, how many terms of the series do we need to add in order to find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$  with error less than  $\frac{1}{60}$ ?

- (a)  $n = 3$
- (b)  $n = 4$
- (c)  $n = 5$
- (d)  $n = 7$
- (e)  $n = 6$

5. (4 pts) Which of the following series converge absolutely?

- (a)  $\sum_{n=1}^{\infty} (-1)^n$
- (b)  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$
- (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
- (d)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$
- (e) All of the above series are absolutely convergent.

6. (4 pts) What is the intersection of the sphere  $(x + 1)^2 + (y - 2)^2 + (z - 3)^2 = 25$  with the  $xz$ -plane?

- (a)  $(x + 1)^2 + (z - 3)^2 = 21$
- (b)  $(x + 1)^2 + (z - 3)^2 = 23$
- (c) The point  $(-1, 2, 3)$
- (d) The point  $(0, 2, 0)$
- (e)  $(x + 1)^2 + (y - 2)^2 = 25$

7. (4 pts) The Interval of Convergence of the series  $\sum_{n=0}^{\infty} \frac{(x + 1)^n (2n + 1)!}{10^n n!}$  is:

- (a)  $I = (-11, 9)$
- (b)  $I = \left(-\frac{1}{10}, \frac{1}{10}\right)$
- (c)  $I = [-11, 9)$
- (d)  $I = (-\infty, \infty)$
- (e)  $I = \{-1\}$

8. (4 pts) Given the triangle with vertices  $A(2, -2, 5)$ ,  $B(1, 1, 4)$  and  $C(3, 1, 3)$ , find the cosine of the angle at  $B$ .

(a)  $\frac{3}{\sqrt{55}}$

(b)  $\frac{1}{\sqrt{55}}$

(c)  $\frac{3}{\sqrt{11}}$

(d)  $\frac{1}{\sqrt{11}}$

(e) None of the above.

9. (4 pts)  $\arctan(x^3) =$

(a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1}$

(b)  $\sum_{n=0}^{\infty} \frac{x^{6n+3}}{2n+1}$

(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{2n+1}$

(d)  $\sum_{n=0}^{\infty} \frac{x^{6n+1}}{6n+1}$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{6n+1}$

10. (4 pts) If we represent  $\frac{1}{9+4x^2}$  as a power series centered at zero, what is the associated radius of convergence?

(a)  $R = \frac{4}{9}$

(b)  $R = \frac{2}{3}$

(c)  $R = \frac{1}{2}$

(d)  $R = \frac{9}{4}$

(e)  $R = \frac{3}{2}$

## PART II WORK OUT

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (12 pts) Find the radius and interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{(-3)^n(2x-1)^n}{n}$

12. (i) (4 pts) Find a Maclaurin Series representaton for  $f(x) = \sin\left(\frac{x^2}{3}\right)$ .

(ii) (6 pts) Using the result in part (i), write  $\int_0^1 \sin\left(\frac{x^2}{3}\right) dx$  as an infinite series.

(iii) (6 pts) Using the series found in part (ii), find  $s_2$ , the sum of the first three nonzero terms, to estimate  $\int_0^1 \sin\left(\frac{x^2}{3}\right) dx$ . Give an upper bound on the error.

13. (10 pts) Does the series  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^4}$  converge absolutely? Justify your answer.

14. (10 pts) Find the Taylor Series for  $f(x) = \ln x$  centered at  $a = 4$ .

15. Let  $f(x) = e^{2-x}$ .

(i) (6 pts) Give the fourth degree Taylor Polynomial for  $f(x)$  centered around  $a = 2$ .

(ii) (6 pts) Use Taylor's Inequality to give a bound on the error when using the polynomial from (i) to estimate  $f(x)$  on the interval  $[-1, 5]$ .

Taylor's Inequality:  $|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ , where  $|f^{(n+1)}(x)| \leq M$  for  $x$  in an interval containing  $a$ .