

Fall 2010 Math 152

Exam I Version B Solutions

1. **D** Integrate by parts with $u = x^2$, $dv = e^x dx$.

Then $\int x^2 e^x dx = x^2 e^x - \int 2x e^x dx$. Integrate by parts again with $u = 2x$, $dv = e^x dx$ yields $x^2 e^x - 2x e^x + \int 2e^x dx = x^2 e^x - 2x e^x + 2e^x + C$.

2. **B** Partition along the y -axis. The slices are square prisms $2x \times 2x \times dy = 4x^2 dy = 4y dy$.

Therefore $V = \int_0^1 4y dy = 2y^2 \Big|_0^1 = 2$.

3. **A** The force (weight) of the rope and weight after it has been pulled up x feet is $F(x) = 200 - 2x$. Therefore, the work done is given by $W = \int_0^{20} (200 - 2x) dx = 200x - x^2 \Big|_0^{20} = 4000 - 400 = 3600$ ft-lbs.

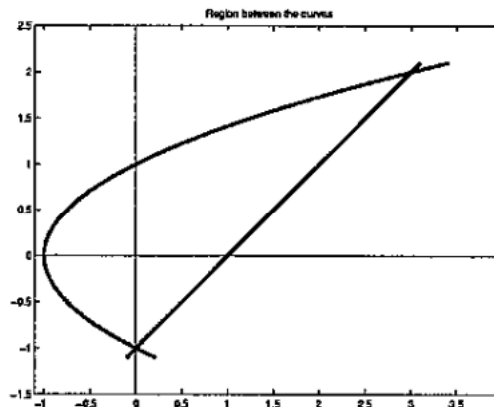
4. **B** $\sec^4 x = \sec^2 x \cdot \sec^2 x = (\tan^2 x + 1) \sec^2 x$. Let $u = \tan x$. Then $du = \sec^2 x dx$. When $x = 0$, $u = 0$, and when $x = \frac{\pi}{4}$, $u = 1$. Substituting into the integral yields $\int_0^1 (u^2 + 1) du = \frac{1}{3}u^3 + u \Big|_0^1 = \frac{4}{3}$.

5. **A** Let $u = 10 + x$. Then $du = dx$. When $x = -1$, $u = 9$, and when $x = 6$, $u = 16$. The integral is therefore equivalent to $\int_9^{16} \frac{x}{\sqrt{u}} du$. Since $u = 10 + x$, $x = u - 10$, so this integral is equivalent to $\int_9^{16} \frac{u - 10}{\sqrt{u}} du = \int_9^{16} (u^{1/2} - 10u^{-1/2}) du$.

6. **B** The sketch of the region is shown below.

Use a partition of the y -axis, so the functions are $x = y + 1$ and $x = y^2 - 1$. These functions intersect when $y + 1 = y^2 - 1$, $y^2 - y - 2 = 0$, $(y - 2)(y + 1) = 0$, or $y = 2, -1$. The area is

therefore $\int_{-1}^2 \left((y + 1) - (y^2 - 1) \right) dy$



7. **E** Using the double-angle identity for $\sin(2x)$

yields $\int_0^{\pi/2} 2 \sin(x) \cos^2(x) dx$. Let $u = \cos(x)$. Then $du = -\sin(x) dx$. When $x = 0$, $u = 1$, and when $x = \frac{\pi}{2}$, $u = 0$. Substituting into the integral yields $-\int_1^0 2u^2 du = \int_0^1 2u^2 du = \frac{2}{3}u^3 \Big|_0^1 = \frac{2}{3}$.

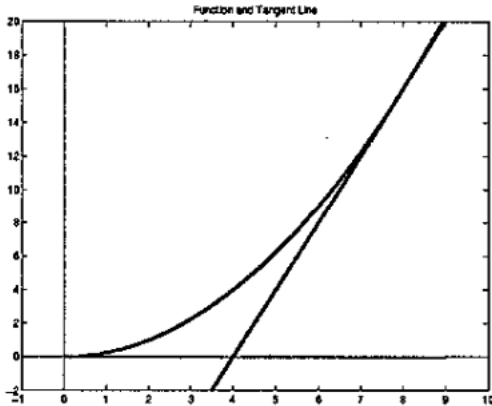
8. **C** Let $u = x^2 + 1$. Then $du = 2x dx$. When $x = 0$, $u = 1$, and when $x = 1$, $u = 2$. Substituting into the integral yields

$\int_1^2 \frac{1}{2} u^{-1/2} du = u^{1/2} \Big|_1^2 = \sqrt{2} - 1$.

9. **D** Create cylindrical shells centered around $x = 2$. The volume of a shell is $V = 2\pi rh dr = 2\pi(2 - x)(x - x^2) dx$. Since the curves intersect at $x = 0$ and $x = 1$, the volume of the solid is $V = 2\pi \int_0^1 (2 - x)(x - x^2) dx$.

10. **A** Integrate by parts with $u = \ln x$, $dv = x^3 dx$. Then $du = \frac{1}{x}$ and $v = \frac{1}{4}x^4$. The integral is equivalent to $\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \left(\frac{1}{x} \right) dx = \frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^3 dx$.

11. The graph is shown below. When $x = 8$, $y = 16$ and $f'(x) = 4$, so the equation of the tangent line is $y = 16 + 4(x - 8) = 4x - 16$. It is easiest to partition the y -axis, so the functions become $x = 2\sqrt{y}$ and $x = \frac{1}{4}(y + 16)$. The area is given by $\int_0^{16} \left(\left(\frac{1}{4}y + 4 \right) - (2\sqrt{y}) \right) dy$. Integrating yields $\frac{1}{8}y^2 + 4y - \frac{4}{3}y^{3/2} \Big|_0^{16} = \left(32 + 64 - \frac{256}{3} \right) = \frac{32}{3}$.



12. .

- (a) Let $u = \tan^{-1} x$ and $dv = x dx$. Then $du = \frac{1}{x^2 + 1} dx$ and $v = \frac{1}{2}x^2$. Then $\int x \tan^{-1} x dx = \frac{1}{2}x^2 \tan^{-1} x - \int \frac{1}{2}x^2 \frac{1}{x^2 + 1} dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{x^2 + 1} dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^2 + 1} \right) dx = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}(x - \tan^{-1} x) + C$.

- (b) $\int \cos^2 x \tan^3 x dx = \int \frac{\sin^3 x}{\cos x} dx = \int \frac{(1 - \cos^2 x) \sin x}{\cos x} dx$. Let $u = \cos x$. Then $du = -\sin x dx$, so the integral is equivalent to $-\int \frac{1 - u^2}{u} du = -\int \left(\frac{1}{u} - u \right) du = -\ln |u| + \frac{1}{2}u^2 + C = -\ln |\cos x| + \frac{1}{2}\cos^2 x + C$.

13. A cross-sectional slice a distance y from the bottom is cylindrical with radius x and height dy . From similar triangles, $\frac{2}{3} = \frac{x}{y}$, so $x = \frac{2}{3}y$. The force (weight) acting on the slice is $F = \rho g \pi \left(\frac{2}{3}y \right)^2 dy$. The distance required to move the slice out of the tank is $4 - y$ (including the spout), so the work required to pump the water out of the tank is $W = \int_0^3 \rho g \pi \cdot \frac{4}{9}y^2(4 - y) dy = \frac{4\rho g \pi}{9} \int_0^3 (4y^2 - y^3) dy = \frac{4\rho g \pi}{9} \left(\frac{4}{3}y^3 - \frac{1}{4}y^4 \right) \Big|_0^3 = \frac{4\rho g \pi}{9} \left(36 - \frac{81}{4} \right) = 7\rho g \pi$ ft-lbs.

14. Slice perpendicular to the x -axis. The slices are washers with outer radius $\sec x$, inner radius $2 \sin x$, and height dx . Therefore, the volume of the solid is $V = \pi \int_0^{\pi/4} (\sec^2 x - 4 \sin^2 x) dx = \pi \int_0^{\pi/4} (\sec^2 x - 2(1 - \cos(2x))) dx = \pi \left(\tan(x) - 2x + \sin(2x) \right) \Big|_0^{\pi/4} = \pi \left(1 - \frac{\pi}{2} + 1 \right) = 2\pi - \frac{\pi^2}{2}$.
15. The average value is found by calculating $\frac{1}{b-0} \int_0^b (3x^2 - 4x - 7) dx = \frac{1}{b}(x^3 - 2x^2 - 7x) \Big|_0^b = b^2 - 2b - 7$. Set this equal to 1 and solve: $b^2 - 2b - 7 = 1$, $b^2 - 2b - 8 = 0$, $(b - 4)(b + 2) = 0$, so $b = 4$ or $b = -2$. Since $b > 0$, our only solution is $b = 4$.