MATH 152, FALL 2010
COMMON EXAM II - VERSION A

LAST NAME, First name (print): ______________________________

INSTRUCTOR: ____________________

SECTION NUMBER: __________

UIN: ______________________________

SEAT NUMBER: ____________________

DIRECTIONS:

1. The use of a calculator, laptop, or computer is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam, as Scantrons will NOT be returned!*
3. In Part 2 (Problems 11-15), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ______________________________

1. The sequence $a_n = \dfrac{1}{5 - e^{-n}}$

(a) converges to $\dfrac{1}{4}$.
(b) converges to 5.
(c) converges to $\dfrac{1}{5}$.
(d) diverges.
(e) converges to 0.

2. The sequence $a_n = \dfrac{(-1)^n(2n^2+2)}{3n^2+1}$

(a) diverges.
(b) converges to $\dfrac{2}{3}$.
(c) converges to $-\dfrac{2}{3}$.
(d) converges to 2.
(e) converges to 0.

3. Which of the following is the form of the partial fraction decomposition of $\dfrac{x+3}{(x+1)^2(x^2+2x+3)}$?

(a) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x^2+2x+3}$
(b) $\dfrac{Ax+B}{(x+1)^2} + \dfrac{C}{x+3} + \dfrac{D}{x-1}$
(c) $\dfrac{A}{x+3} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{x-1} + \dfrac{E}{x+3}$
(d) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{Cx+D}{x^2+2x+3}$
(e) $\dfrac{A}{x+1} + \dfrac{Bx+C}{(x+1)^2} + \dfrac{D}{x^2+2x+3}$

4. The nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = \dfrac{2n-1}{n+1}$. Which of the following statements is true?

I. The series $\sum_{n=1}^{\infty} a_n$ converges to 2

II. The series $\sum_{n=1}^{\infty} a_n$ diverges by the Test for Divergence

III. The sequence a_n converges to 0

(a) only I and III are true

(b) only I is true

(c) only II and III are true

(d) only II is true

(e) all three statements are true

5. Which of the following integrals gives the area of the surface obtained by rotating the curve $y = e^{2x}$, $0 \leq x \leq 1$ about the x-axis?

(a) $\displaystyle\int_0^1 2\pi x\sqrt{1+4e^{4x}}\,dx$

(b) $\displaystyle\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}}\,dx$

(c) $\displaystyle\int_0^1 2\pi x\sqrt{1+\frac{1}{4}e^{4x}}\,dx$

(d) $\displaystyle\int_0^1 2\pi\sqrt{1+2e^{2x}}\,dx$

(e) $\displaystyle\int_0^1 2\pi e^{2x}\sqrt{1+\frac{1}{4}e^{4x}}\,dx$

6. After an appropriate substitution, the integral $\int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$ is equivalent to which of the following?

(a) $4\int_{\pi/6}^{\pi/2} \sec\theta \tan^2\theta\,d\theta$

(b) $2\int_{\pi/3}^{\pi/2} \cos\theta\,d\theta$

(c) $4\int_{\pi/6}^{\pi/2} \cos^2\theta\,d\theta$

(d) $2\int_{\pi/6}^{\pi/2} \tan\theta\,d\theta$

(e) $4\int_{\pi/3}^{\pi/2} \cos^2\theta\,d\theta$

7. Compute $\int_{-1}^{3} \frac{1}{x^2}\,dx$.

(a) $\frac{4}{3}$

(b) The integral diverges

(c) $\ln 9$

(d) $-\frac{4}{3}$

(e) $\frac{2}{3}$

8. Which statement is true about the integral $\int_{1}^{\infty} \frac{\sin^2 x}{x^2}\,dx$?

(a) The integral diverges by oscillation.

(b) The integral converges to 0.

(c) The integral diverges by comparison to $\int_{1}^{\infty} \frac{1}{x}\,dx$.

(d) The integral diverges by comparison to $\int_{1}^{\infty} 1^2\,dx$.

(e) The integral converges by comparison to $\int_{1}^{\infty} \frac{1}{x^2}\,dx$.

9. Which statement is true about the series $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^3-5}$?

(a) The series is divergent because $\dfrac{n}{n^3-5} > \dfrac{1}{n}$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}$ is divergent.

(b) The series is convergent because $\dfrac{n}{n^3-5} < \dfrac{1}{n^2}$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ is convergent.

(c) The series is convergent because $\displaystyle\lim_{n\to\infty} \frac{n}{n^3-5} = 0$.

(d) The series is divergent because $\displaystyle\lim_{n\to\infty} \frac{n}{n^3-5} \neq 0$.

(e) The series is convergent because $\displaystyle\lim_{n\to\infty} \frac{\frac{n}{n^3-5}}{\frac{1}{n^2}} = 1$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ is convergent.

10. Which of the following series is convergent?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

(e) More than one of these series is convergent.

PART II WORK OUT

<u>Directions</u>: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (10 points) Compute $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

12. (8 points each) Find the sum of the following series or show they are divergent:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2+2^n}{10^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{(n+1)(n+3)}$

13. (6 points each) Given the curve parametrized by
$x = \cos t + t \sin t,\ y = \sin t - t \cos t,\ 0 \leq t \leq \frac{\pi}{2}$:

(a) Find the length of the curve.

(b) SET UP, BUT DO NOT EVALUATE, an integral to find the area of the surface formed by rotating the curve about the y-axis.

14. (10 points) Compute $\displaystyle\int \frac{3x^2 - 4x + 11}{(x-1)(x^2+4)}\,dx$

15. (6 points each) Given the series $\sum_{n=1}^{\infty} 3n^2 e^{-n^3}$:

(a) Use the Integral Test to show that the series is convergent.

(b) According to Matlab, the third partial sum $s_3 \approx 1.107663875$. Use the remainder theorem to estimate the largest possible error.

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-10		40
11		10
12		16
13		12
14		10
15		12
TOTAL		100