

**MATH 152, FALL 2010
COMMON EXAM III - VERSION A**

LAST NAME, First name (print): _____

INSTRUCTOR: _____

SECTION NUMBER: _____

UIN: _____

SEAT NUMBER: _____

DIRECTIONS:

1. The use of a calculator, laptop, or computer is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your ScanTron using a No. 2 pencil. *For your own records, also record your choices on your exam, as Scantrons will NOT be returned!*
3. In Part 2 (Problems 11-15), present your solutions in the space provided. *Show all your work neatly and concisely and clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to *write your name, section and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: _____

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-10		40
11		10
12		14
13		12
14		14
15		10
TOTAL		100

1. Find the cosine of the angle between the vectors $\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} + 4\vec{j} + 3\vec{k}$.

(a) $\frac{8}{\sqrt{41}}$

(b) $\frac{4}{\sqrt{75}}$

(c) $\frac{4}{\sqrt{26}}$

(d) $\frac{8}{\sqrt{53}}$

(e) $\frac{8}{\sqrt{78}}$

2. Find the coefficient of x^6 in the Maclaurin series for $f(x) = \frac{1}{1 + 4x^2}$.

(a) 256

(b) 0

(c) 64

(d) -256

(e) -64

3. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{4^n}{n!} (x - 2)^n$.

(a) $\frac{1}{4}$

(b) ∞

(c) 0

(d) 4

(e) $\frac{9}{4}$

4. Given the series $\sum_{n=1}^{\infty} (-1)^n 3n^2 e^{-n^3}$, estimate the error in using the 3rd partial sum (s_3) to approximate the total sum.

- (a) $48e^{-64} - 27e^{-27}$
- (b) e^{-27}
- (c) $48e^{-64}$
- (d) $27e^{-27}$
- (e) e^{-64}

5. Which of the following is the Maclaurin series for $f(x) = e^{-x^2}$.

- (a) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$
- (b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$
- (c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
- (d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$
- (e) $\sum_{n=0}^{\infty} \frac{-x^{n+2}}{n!}$

6. Which statement is true about the convergence of the following two series?

(I) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n+2)3^n}{2^{2n+1}}$ (II) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n^3+4)}{2n^2}$

- (a) Both are absolutely convergent
- (b) (I) is absolutely convergent, (II) is divergent
- (c) Both are divergent
- (d) Both are convergent, but not absolutely convergent
- (e) (I) is divergent, (II) is absolutely convergent

7. Given the power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 4$, which of the following statements is certain to be true?

(a) $\sum_{n=0}^{\infty} c_n (-2)^n$ is convergent

(b) $\sum_{n=0}^{\infty} c_n (-6)^n$ is divergent

(c) $\sum_{n=0}^{\infty} c_n 9^n$ is convergent

(d) $\sum_{n=0}^{\infty} c_n (-4)^n$ is divergent

(e) None of these statements are certain to be true

8. Which of the following is a power series representation for $f(x) = \ln(1+x)$. (HINT: find a power series representation for $f'(x)$ first)

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

(b) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!}$

(d) $\sum_{n=0}^{\infty} (-1)^n x^{n+1}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

9. Find the center and radius of the sphere whose equation is $x^2 + 2x + y^2 - 4y + z^2 = 0$

- (a) center $(-1, 2, 0)$, radius $= 5$
- (b) center $(-1, 2, 0)$, radius $= \sqrt{5}$
- (c) center $(1, -2, 0)$, radius $= \sqrt{5}$
- (d) center $(1, -2, 0)$, radius $= 5$
- (e) center $(-1, 2, 0)$, radius $= 2\sqrt{5}$

10. Find all values of t such that the vectors $\vec{\mathbf{a}} = \langle t + 2, t, t \rangle$ and $\vec{\mathbf{b}} = \langle t - 2, t + 1, t \rangle$ are orthogonal.

- (a) $t = -\frac{1}{6}$
- (b) $t = -\frac{3}{2}, t = -1$
- (c) $t = 1$
- (d) $t = -\frac{4}{3}, t = 1$
- (e) $t = -2, t = 1$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (10 points) Find the Taylor Series for $f(x) = \cos x$ centered at $a = \frac{\pi}{2}$.

12. (7 points each)

- (a) Find the third degree Taylor Polynomial ($T_3(x)$) for the function $f(x) = \sqrt{x}$ at the number $a = 1$.

- (b) Use Taylor's Inequality (see below) to estimate the accuracy of the approximation $f(x) \approx T_3(x)$ when $1 \leq x \leq 2$.

If $|f^{(n+1)}(x)| \leq M$ for all x in a given interval, then $|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

13. (6 points each)

(a) Explain in detail why the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 4}$ is convergent.

(b) Determine whether the series above is absolutely convergent or not. Clearly explain your reasoning.

14. The vertices of a triangle are $A(2, 4, 5)$, $B(3, 5, 3)$, and $C(2, 8, -3)$.

(a) (10 pts total) Let $\vec{\mathbf{a}}$ be the vector from A to B , $\vec{\mathbf{b}}$ be the vector from A to C , and $\vec{\mathbf{c}} = \text{proj}_{\vec{\mathbf{b}}}\vec{\mathbf{a}}$ (the vector projection of \mathbf{a} onto \mathbf{b}). Compute the following:

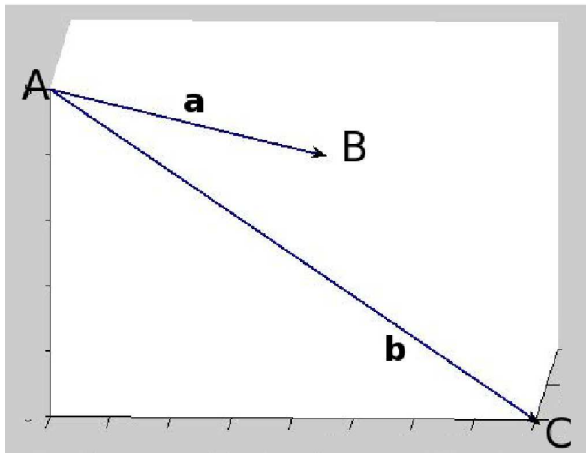
i. (2 pts) $\vec{\mathbf{a}} =$

ii. (2 pts) $\vec{\mathbf{b}} =$

iii. (4 pts) $\vec{\mathbf{c}} =$

iv. (2 pts) $\vec{\mathbf{a}} - \vec{\mathbf{c}} =$

(b) (4 pts) The vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are shown below. Sketch the vector $\vec{\mathbf{c}}$ and the vector $\vec{\mathbf{a}} - \vec{\mathbf{c}}$.



15. (10 points) Find the radius of convergence and interval of convergence of the power

series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{(n+2)4^n}$.

DO NOT WRITE BELOW!

Question	Points Awarded	Points
1-10		40
11		10
12		14
13		12
14		14
15		10
TOTAL		100