EXAM A
QUESTION 1

$$\int_1^2 \ln(x)\,dx \qquad u = \ln(x) \quad v = x \qquad du = \frac{1}{x}dx \quad dv = dx$$

BY PARTS.

$$= x\ln(x)\Big|_1^2 - \int_1^2 x \cdot \frac{1}{x}dx$$

$$= 2\ln(2) - 1 = C$$

$\boxed{2}$ CHAIN RULE + FIRST FTOC

$$F(x) = G(h(x))$$

WHERE $G(u) = \int_0^u e^{t^2}\,dt$

$h(x) = \sin(x)$

$$F'(x) = G'(h(x))\,h'(x)$$

$$G'(u) = e^{u^2} \quad \text{(FTOC)}$$

$$F'(x) = e^{\sin^2(x)}\cos(x)$$

$$= C$$

(3) $\int_0^1 \frac{3x}{\sqrt[3]{x^2+1}}\,dx$

SUBSTITUTION

$u = x^2+1$

$du = 2x\,dx$

$= \int_1^2 \frac{3}{2} u^{-1/3}\,du = \frac{3}{2}\left(\frac{3}{2}u^{2/3}\right)\Big|_1^2$

$= \frac{9}{4}\left(\sqrt[3]{4}-1\right) =$ (A)

(4) $\frac{1}{\pi/2}\int_0^{\pi/2} \cos^2(x)\sin(2x)\,dx$

$= \frac{2}{\pi}\int_0^{\pi/2} 2\cos^3(x)\sin(x)\,dx$

$= \frac{4}{\pi}\left(-\frac{\cos^4(x)}{4}\right)\Big|_0^{\pi/2} = \frac{1}{\pi}$

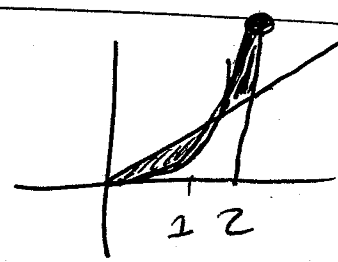
SUBSTITUTION

w/ $u=\cos(x)$

$du = -\sin(x)\,dx$

$=$ (B)

(5) $x^3 = x$ AT $x = 0, 1$.



$\int_1^2 (x^3-x)\,dx + \int_0^1 (x-x^3)\,dx$

$= \frac{x^4}{4} - \frac{x^2}{2}\Big|_1^2 + \left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Big|_0^1$

$= 2 + \frac{1}{4} + \frac{1}{4} + 0 = 5/2 =$ (E)

(6) $\int_{-1}^{4} \frac{x}{(5+x)^2}\,dx$

$u = 5 + x$
$u - 5 = x$
$du = dx$

$= \int_{4}^{9} \frac{u-5}{u^2}\,du$

$= \int_{4}^{9} u^{-1} - 5u^{-2} =$ (D)

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(7) ROPE → $\frac{1}{5}$ N/m , 30 m LONG.

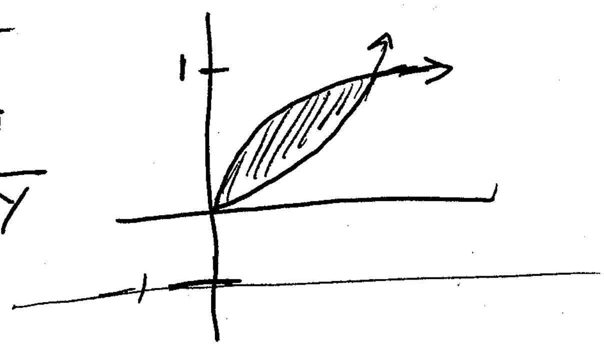
$\int_{0}^{30} \left(15 + \frac{1}{5}x\right)dx$

$= 15x + \frac{x^2}{10}\Big|_{0}^{30}$

$= 450 + 90 = 540 =$ (D)

(8) $y = \sqrt{x} \rightarrow x = y^2$

$y = x^2 \rightarrow$ $x = \sqrt{y}$



FOR $0 \le x \le 1$

VOLUME $= \int_{0}^{1} 2\pi \underbrace{(y+1)}_{\text{RADIUS OF SHELL}} \underbrace{(\sqrt{y} - y^2)}_{\text{HEIGHT OF SHELL}}\,dy$

= (B)

(9) $\int_0^{\pi/4} x\cos(x)\,dx$

BY PARTS

$u = x$	$v = \sin(x)$
$du = dx$	$dv = \cos(x)dx$

$$= x\sin(x)\Big|_0^{\pi/4} - \int_0^{\pi/4} \sin(x)\,dx$$

$$= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) + \cos(x)\Big|_0^{\pi/4}$$

$$= \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1 = \text{(A)}$$

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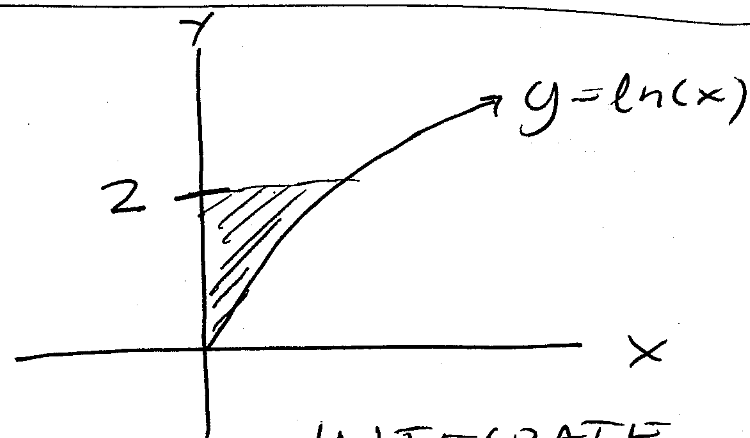
(10) $y = \ln(x)$

$\rightarrow x = e^y$



$$\int_0^2 \pi(e^y)^2\,dy$$

$$= \frac{\pi e^{2y}}{2}\Big|_0^2$$

INTEGRATE WITH RESPECT TO Y!

$$= \frac{\pi e^4}{2} - \frac{\pi}{2} = \text{(C)}$$

11a $\int x^3 \ln(x)\,dx$

BY PARTS

$u = \ln(x)$ $\quad v = \frac{x^4}{4}$

$du = \frac{1}{x}dx$ $\quad dv = x^3 dx$

$$= \frac{\ln(|x|)x^4}{4} - \int \frac{x^3}{4}\,dx$$

$$= \frac{\ln|x|\,x^4}{4} - \frac{x^4}{16} + C$$

11b $\int \cos^3 x \sin^2(x)\,dx$

$$= \int \cos^2(x)\sin^2(x)\cos(x)\,dx$$

$$= \int (1-\sin^2(x))\sin^2(x)\cos(x)\,dx$$

$$= \int (u^2 - u^4)\,du$$

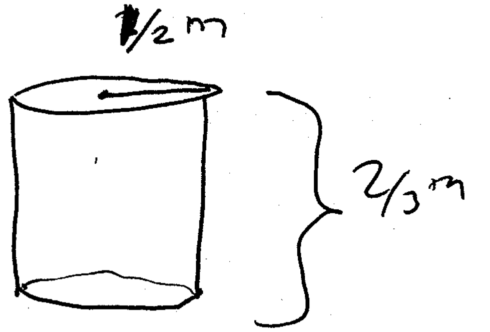
$u = \sin(x)$

$du = \cos(x)dx$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3(x)}{3} - \frac{\sin^5(x)}{5} + C$$

(12)



x = DISTANCE TO TOP

$$W = \int_0^{2/3} \underbrace{\rho g A(x)}_{\text{AREA OF CROSS SECTION}} \underbrace{x}_{\text{DISTANCE TO MOVE CROSS SECTION}} \, dx$$

$$= \int_0^{2/3} \rho g \pi \left(\tfrac{1}{2}\right)^2 x \, dx$$

$$= \frac{\pi \rho g}{4} \left. \frac{x^2}{2} \right|_0^{2/3} = \frac{\pi \rho g}{18}$$

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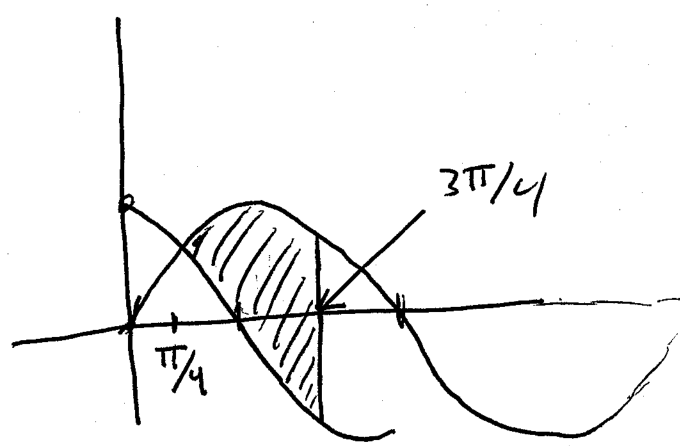
(13)

LENGTH OF VERTICAL CROSS SECTION AT x
$= \sin(x) - \cos(x)$



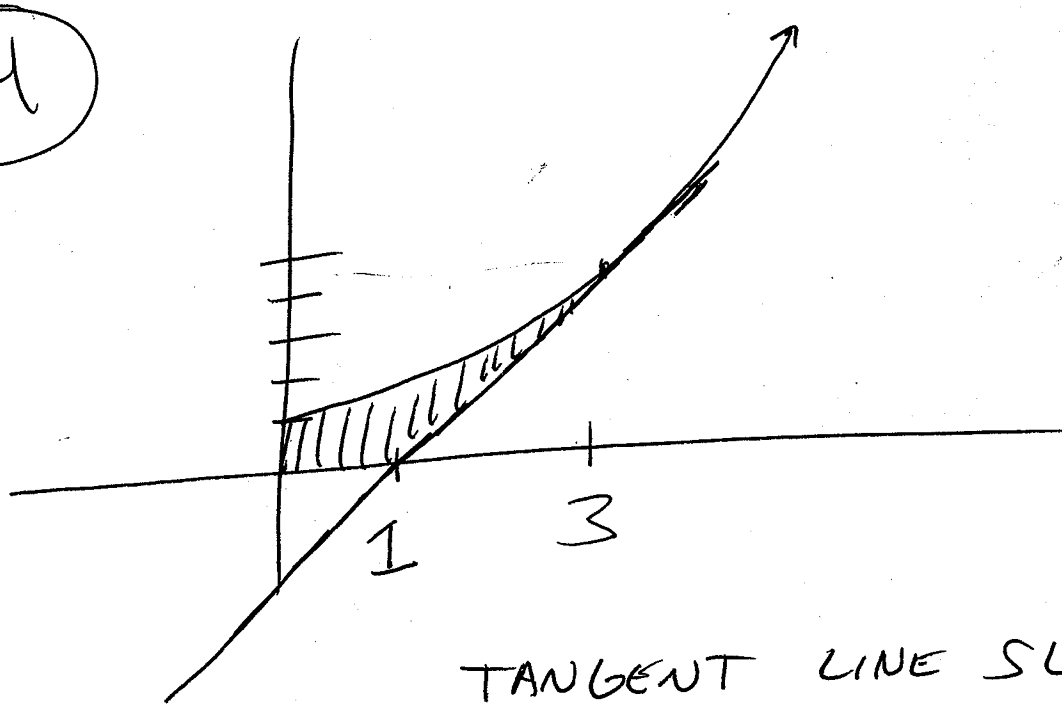
$$A(x) = (\sin(x) - \cos(x))^2$$

$$\int_{\pi/4}^{3\pi/4} (\sin(x) - \cos(x))^2 = \int_{\pi/4}^{3\pi/4} 1 - 2\sin x \cos x \, dx$$

$$= \int_{\pi/4}^{3\pi/4} 1 - \sin 2x \, dx = \left. x + \frac{\cos 2x}{2} \right|_{\pi/4}^{3\pi/4}$$

$$= \pi/2$$

(14)



TANGENT LINE SLOPE

$$= \frac{d}{dx}\left(e^{x/2}\right)\Big|_{x=3} = \frac{1}{2}e^{3/2}$$

TANGENT LINE $\Rightarrow$ $y = \frac{1}{2}e^{3/2}x - \frac{1}{2}e^{3/2}$

x-INTERCT, $x = 1$.

$$\int_1^3 \left(e^{x/2} - \left(\frac{1}{2}e^{3/2}x - \frac{1}{2}e^{3/2}\right)\right)dx + \int_0^1 e^{x/2}\,dx$$

$$= 2e^{x/2} - \frac{e^{3/2}x^2}{4} + \frac{e^{3/2}x}{2}\Big|_1^3 + 2e^{x/2}\Big|_0^1$$

$$= 2e^{3/2} - \frac{9e^{3/2}}{4} + \frac{3e^{3/2}}{2}$$

$$- 2e^{1/2} + \frac{e^{3/2}}{4} + \frac{e^{3/2}}{2}$$

$$+ 2e^{1/2} - 2$$

$$= -2 + 2e^{3/2}$$

(15) $\int x^2 \sin(2\pi x)\,dx$

BY PARTS ... TWICE!

$u = x^2 \qquad v = \frac{-\cos(2\pi x)}{2\pi}$

$du = 2x\,dx \qquad dv = \sin(2\pi x)\,dx$

$$= \frac{-x^2\cos(2\pi x)}{2\pi} + \int \frac{x\cos(2\pi x)}{\pi}\,dx$$

$u = x \qquad v = \frac{\sin(2\pi x)}{2\pi}$

$du = dx \qquad dv = \cos(2\pi x)\,dx$

$$= -\frac{x^2\cos(2\pi x)}{2\pi} + \frac{x\sin(2\pi x)}{2\pi^2} - \int \frac{\sin(2\pi x)}{2\pi^2}\,dx$$

$$= \frac{-x^2\cos(2\pi x)}{2\pi} + \frac{x\sin(2\pi x)}{2\pi^2} + \frac{\cos(2\pi x)}{4\pi^3} + C$$