EXAM B

Q 1

CHANGE INTERVAL TO $[0, \pi/2]$

$$\frac{1}{\pi/2}\int_0^{\pi/2} -\cos^2(x)\sin(2x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2} -\cos^2(x)\,2\sin(x)\cos(x)\,dx$$

$u = \cos(x)$

$du = -\sin(x)dx$

$$= \int_1^0 \frac{4}{\pi} u^3\,du$$

$$= \frac{4u^4}{4\pi}\Big|_1^0 = -\frac{1}{\pi} = \text{B}$$

Q2 ROPE IS $\frac{1}{5}\frac{\text{NEWTONS}}{\text{METER}}$

$$\int_0^{20} \left(15 + \frac{x}{5}\right)dx$$

$$= 15x + \frac{x^2}{10}\Big|_0^{20}$$

$$= 300 + 40 = 340 = \text{D}$$

③ CONSIDER $F(x) = \int_0^{\cos x} e^{t^2} dt$

$= G(h(x))$

WHERE $G(u) = \int_0^u e^{t^2} dt$

$h(x) = \cos(x)$

FTOC(1) $\longrightarrow$ $G'(u) = e^{u^2}$

CHAIN RULE $\longrightarrow$ $F'(x) = G'(h(x))\,h'(x)$

$= e^{\cos^2(x)}(-\sin(x))$

= B

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④ $\int_0^2 \frac{5x}{(x^2+1)^{1/3}}$ $\boxed{\text{SUBSTITUTION}}$ $u = x^2+1$

$du = 2x\,dx$

$= 5\int_1^5 \frac{u^{-1/3}\,du}{2} = \frac{5}{2}\left(\frac{3}{2}u^{2/3}\right)\Big|_1^5$

$= 15/4\,(\sqrt[3]{25} - 1)$

= E

$\boxed{5}\ \int_1^3 \ln(x)\,dx$

BY PARTS

$u = \ln(x) \qquad v = x$

$du = \frac{1}{x}dx \qquad dv = dx$

$= x\ln(x)\Big|_1^3$

$- \int_1^3 x\left(1/\cancel{x}\right)dx$

$= 3\ln(3) - 2 = A$

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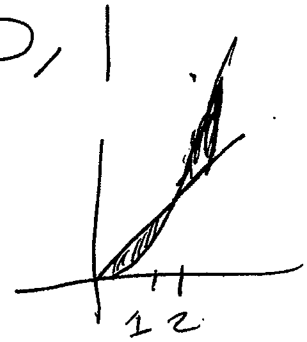
6 $\quad x^2 = x$ AT $x = 0, 1$

AREA $= \int_1^2 x^2 - x + \int_0^1 x - x^2$



$= \left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2 + \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1$

$= \frac{8}{3} - \frac{4}{2} - \frac{1}{3} + \frac{1}{2} + \frac{1}{2} - \frac{1}{3}$

$= 1 = E$

7 $\displaystyle\int_{-3}^{5} \frac{x}{(4+x)^2}\,dx$

$$\begin{cases} u = 4 + x \\ du = dx \\ u - 4 = x \end{cases}$$

$$= \int_1^9 \frac{u-4}{u^2}\,du$$

$$= \int_1^9 u^{-1} - 4u^{-2}\,du = B$$

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8 $\displaystyle\int_0^{\pi/4} x\cos(x)\,dx$

BY PARTS

$u = x \qquad v = \sin(x)$

$du = dx \qquad dv = \cos(x)\,dx$

$$= x\sin(x)\Big|_0^{\pi/4} - \int_0^{\pi/4} \sin(x)\,dx$$

$$= x\sin(x) + \cos(x)\Big|_0^{\pi/4}$$

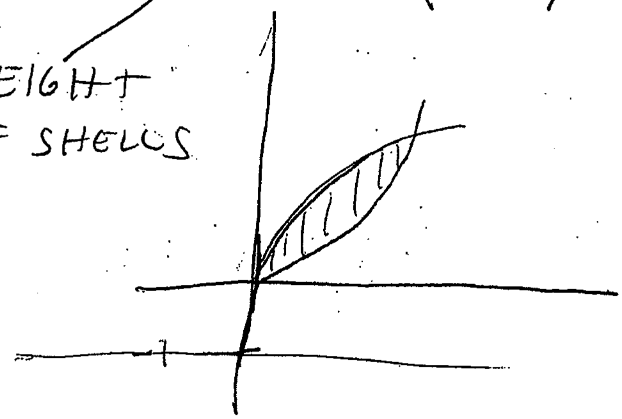
$$= \frac{\pi}{4}\left(\frac{2}{\sqrt{2}}\right) + \frac{2}{\sqrt{2}} - 1 = D$$

(9) SWITCH FUNCTIONS TO $y=x^2$, $y=\sqrt[3]{x}$.

$$\text{VOLUME} = \int_0^1 2\pi \underbrace{(y+1)}_{\text{RADIUS OF ROTATION}} \underbrace{(\sqrt{y}-y^3)}_{\text{HEIGHT OF SHELLS}}\,dy$$

$x = y^3$

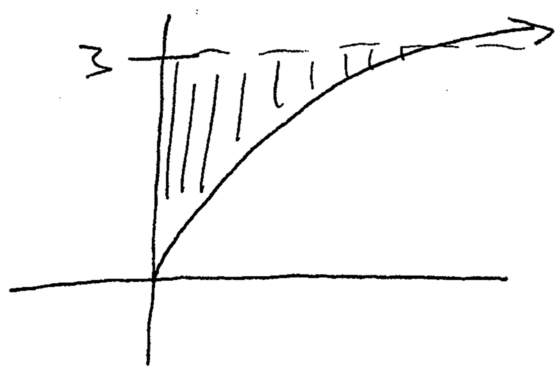
$x = \sqrt{y}$



$= \text{C}$

(10) $y = \ln(x)$

$\rightarrow e^y = x$



$$\int_0^3 \pi (e^y)^2\,dy$$

$$= \frac{\pi e^{2y}}{2}\Big|_0^3 = \frac{\pi}{2}(e^6-1)$$

$= \text{D}$

$\boxed{11a}$ $\int x^2 \cos(2\pi x)\,dx$ — BY PARTS TWICE

$$= \frac{x^2 \sin(2\pi x)}{2\pi} - \int \frac{x \sin(2\pi x)}{\pi}\,dx$$

$u = x^2 \quad v = \frac{\sin(2\pi x)}{2\pi}$

$du = 2x\,dx \quad dv = \cos(2\pi x)\,dx$

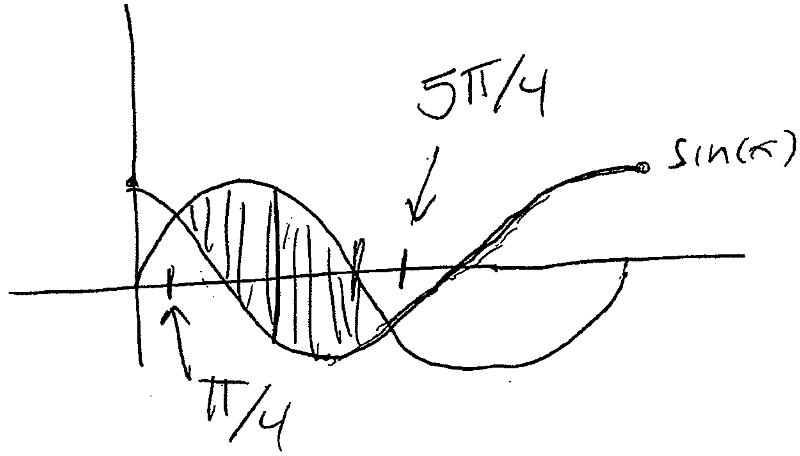
$u = x \quad v = \frac{-\cos(2\pi x)}{2\pi^2}$

$du = dx \quad dv = \frac{\sin(2\pi x)}{\pi}\,dx$

$$= \frac{x^2 \sin(2\pi x)}{2\pi} + \frac{x \cos(2\pi x)}{2\pi^2} - \int \frac{\cos(2\pi x)}{2\pi^2}\,dx$$

$$= \frac{x^2 \sin(2\pi x)}{2\pi} + \frac{x \cos(2\pi x)}{2\pi^2} - \frac{\sin(2\pi x)}{4\pi^3} + C$$

(12)



LENGTH OF VERTICAL CROSS SECTIONS $= \sin(x) - \cos(x)$

$$\int_{\pi/4}^{5\pi/4} \underbrace{\left(\sin(x) - \cos(x)\right)^2}_{\text{AREA OF SQUARE AT } x} dx$$

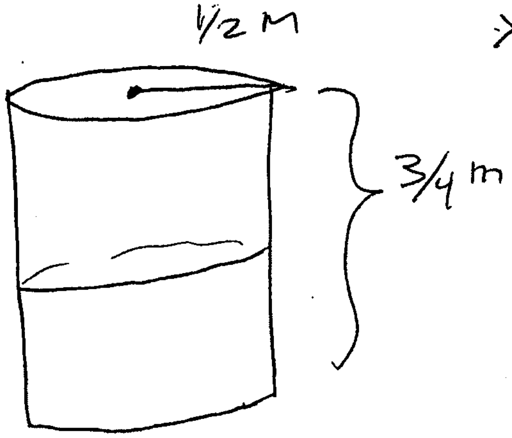
$$= \int_{\pi/4}^{5\pi/4} \left(1 - 2\sin(x)\cos(x)\right) dx$$

$$= \int_{\pi/4}^{5\pi/4} \left(1 - \sin(2x)\right) dx$$

$$= x + \frac{\cos(2x)}{2} \Big|_{\pi/4}^{\frac{5\pi}{4}}$$

$$= \pi$$

(13)



x = DISTANCE TO THE TOP

AREA OF CIRCULAR CROSS SECTION

$= \pi \left(\frac{1}{2}\right)^2$

$= \pi/4$

$$W = \int_0^{3/4} \rho g \underbrace{\left(\frac{\pi}{4}\right)}_{\text{AREA OF CROSS SECTIONS}} x \, dx$$

$$= \frac{\rho g \pi}{4} \left(\frac{x^2}{2}\right)\Big|_0^{3/4} = \frac{9 \rho g \pi}{128}$$

$\boxed{14a}$ $\int x^4 \ln(x)\,dx$

$u = \ln(x)$ $v = \frac{x^5}{5}$

$du = \frac{1}{x}dx$ $dv = x^4 dx$

$$= \frac{x^5 \ln(x)}{5} - \int \frac{x^4}{5}\,dx$$

$$= \frac{x^5 \ln(x)}{5} - \frac{x^5}{25} + C$$

$\boxed{14b}$ $\int \cos^2(x)\sin^3(x)\,dx$

$$= \int \cos^2(x)\left(1-\cos^2(x)\right)\sin(x)\,dx$$

$u = \cos(x)$

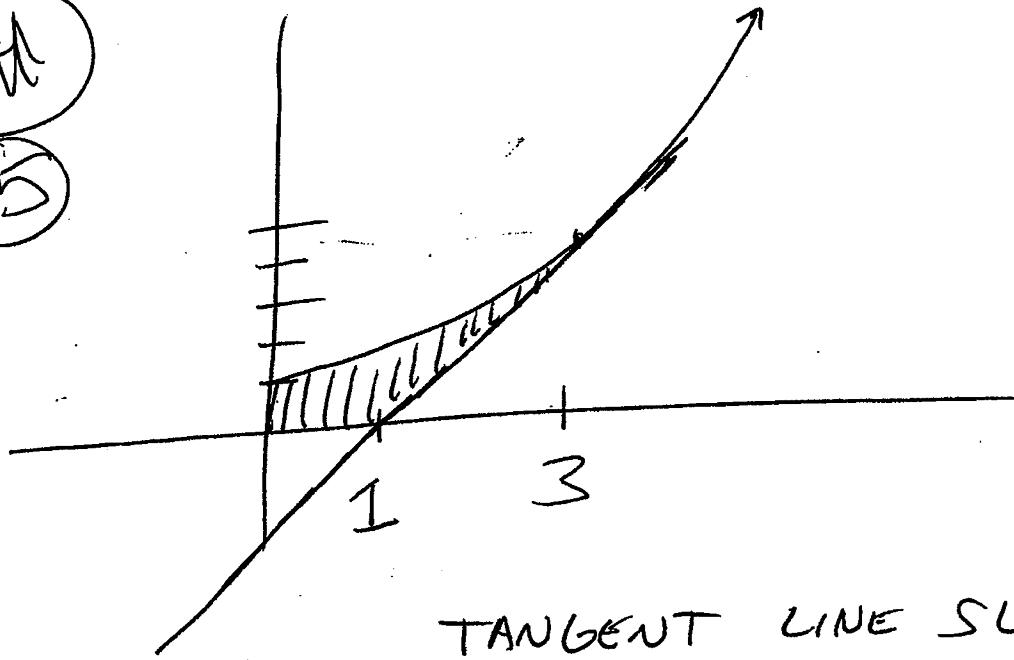
$du = -\sin(x)\,dx$

$$= -\int \left(u^2 - u^4\right)du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + C$$

15



TANGENT LINE SLOPE

$$= \frac{d}{dx}\left(e^{x/2}\right)\Big|_{x=3} = \frac{1}{2}e^{3/2}$$

TANGENT LINE $\Rightarrow$ $y = \frac{1}{2}e^{3/2}x - \frac{1}{2}e^{3/2}$

X-INTERCT , $x = 1$.

$$\int_1^3 \left(e^{x/2} - \left(\frac{1}{2}e^{3/2}x - \frac{1}{2}e^{3/2}\right)\right)dx + \int_0^1 e^{x/2}\,dx$$

$$= 2e^{x/2} - \frac{e^{3/2}x^2}{4} + \frac{e^{3/2}x}{2}\Big|_1^3 + 2e^{x/2}\Big|_0^1$$

$$= 2e^{3/2} - \frac{9e^{3/2}}{4} + \frac{3e^{3/2}}{2}$$

$$- 2e^{1/2} + \frac{e^{3/2}}{4} + \frac{e^{3/2}}{2}$$

$$+ 2e^{1/2} - 2$$

$$= -2 + 2e^{3/2}$$