PRINT

LAST NAME Solutions FIRST NAME Math 152 Instructors

INSTRUCTOR: All SECTION NUMBER: All

UIN: SEAT NUMBER: 0

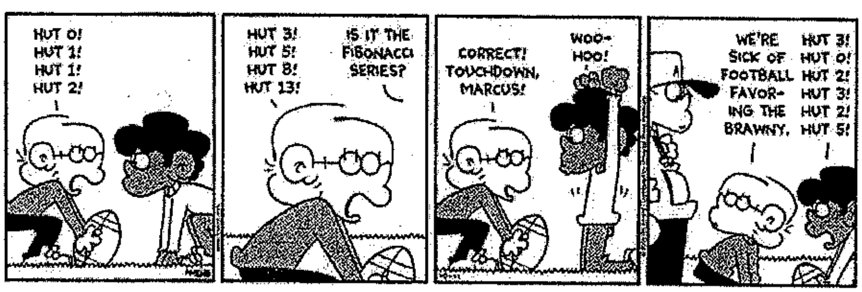
Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 11-15), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section and version letter of the exam on the Scantron form.**
5. Good Luck!

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: Instructors



http://foxtrot.com

Question	1-10	11	12	13	14	15-16	TOTAL
Points Awarded	50	10	10	10	10	10	100
Points Possible	50	10	10	10	10	10	100

1. The partial fraction decomposition of $\dfrac{x+2}{(x^2-5x-6)(x-1)^2(x^2+3)}$ is

$x^2-5x-6=(x-6)(x+1)$

(a) $\dfrac{A}{x^2-5x-6}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}+\dfrac{D}{x^2+3}$

(b) $\dfrac{Ax+B}{x^2-5x-6}+\dfrac{C}{x-1}+\dfrac{D}{x^2+3}$

(c) $\dfrac{A}{x-6}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{Dx+E}{x^2+3}$

(d) $\dfrac{A}{x-6}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{D}{(x-1)^2}+\dfrac{Ex+F}{x^2+3}$

(e) $\dfrac{A}{x-6}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{Dx+E}{(x-1)^2}+\dfrac{Fx+G}{x^2+3}$

$= \dfrac{A}{x-6}+\dfrac{B}{x+1}+\dfrac{C}{x-1}+\dfrac{D}{(x-1)^2}+\dfrac{Ex+F}{x^2+3}$

2. Which of the following integrals gives the area of the surface obtained by rotating the curve $y=e^{x/2}$ $0\le x\le 2$ about the x-axis.

$SA = 2\pi\int_0^2 y\sqrt{1+(y')^2}\,dx$

$y=e^{x/2}$ $\quad y'=\frac{1}{2}e^{x/2}$ $\quad (y')^2=\frac{1}{4}e^x$

$2\pi\int_0^2 e^{x/2}\sqrt{1+\frac{1}{4}e^x}\,dx$

(a) $2\pi\displaystyle\int_0^2 e^{x/2}\sqrt{1+\frac{e^x}{4}}\,dx$

(b) $2\pi\displaystyle\int_0^2 x\sqrt{1+\frac{e^x}{4}}\,dx$

(c) $2\pi\displaystyle\int_0^1 x\sqrt{1+e^{x/2}}\,dx$

(d) $2\pi\displaystyle\int_0^2 x\sqrt{1+4e^x}\,dx$

(e) $2\pi\displaystyle\int_0^2 e^{x/2}\sqrt{1+4e^x}\,dx$

3. Compute the arc length of the curve given by the parametric equations $x=\left(\frac{\sqrt{2}}{3}\right)t^{3/2}$, $y=t+27$ from $t=0$ to $t=6$

(a) $\dfrac{28}{3}$

(b) $\dfrac{1}{3}(13^{3/2}-1)$

(c) $\dfrac{14}{3}$

(d) $\dfrac{4}{3}(6)^{3/2}$

(e) $\dfrac{32}{3}$

$S=\int_0^6\sqrt{(x')^2+(y')^2}\,dt$ $\quad x'=\left(\frac{\sqrt{2}}{3}\right)\left(\frac{3}{2}\right)t^{1/2}$

$y'=1$

$(x')^2=\frac{2}{4}t=\frac{1}{2}t$

$S=\int_0^6\sqrt{\frac{1}{2}t+1}\,dt = 2\int_1^4 u^{1/2}\,du$

$u=\frac{1}{2}t+1$ $\quad du=\frac{1}{2}dt$

$t=0$ $u=1$ $\quad t=6$ $u=4$

$=2\left(\frac{2}{3}\right)\left[u^{3/2}\right]_1^4$

$=\frac{4}{3}[8-1]=\frac{28}{3}$

4. Which of the following integrals is equivalent to $\int_{\sqrt{3}}^{3} \frac{\sqrt{x^2+9}}{x}\,dx$?

(a) $\int_{\pi/3}^{\pi/2} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$

(b) $\int_{\pi/6}^{\pi/4} \frac{\sec(\theta)}{\tan(\theta)}\,d\theta$

(c) $3\int_{\pi/3}^{\pi/4} \frac{\sec(\theta)}{\tan(\theta)}\,d\theta$

(d) $9\int_{\pi/3}^{\pi/2} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$

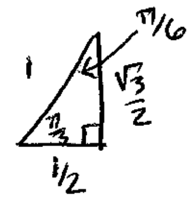
(e) $3\int_{\pi/6}^{\pi/4} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$

$x = 3\tan\theta$ $\quad x=3 \quad \tan\theta = 1$

$dx = 3\sec^2\theta\,d\theta \quad \theta = \pi/4$

$x = \sqrt{3} \quad \tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$



$$\int_{\pi/6}^{\pi/4} \frac{\sqrt{9\tan^2\theta + 9}}{3\tan\theta}\, 3\sec^2\theta\,d\theta = 3\int_{\pi/6}^{\pi/4} \frac{\sec^3\theta}{\tan\theta}\,d\theta$$

5. The integral $\int_{-1}^{1} \frac{1}{x^2}\,dx$

(a) Diverges

(b) Converges to -2

(c) Converges to 2

(d) Converges to 0

(e) Converges to -1

$$\int_{-1}^{0} x^{-2}\,dx + \int_{0}^{1} x^{-2}\,dx = \lim_{t\to 0^-}\left[-x^{-1}\right]_{-1}^{t} + \lim_{t\to 0^+}\left[-x^{-1}\right]_{t}^{1}$$

$$= +\infty + (-1) - 1 + \infty \quad \text{Diverges}$$

6. The sequence $a_n = \frac{3n^2+2n+1}{5-7n^2}$ for $n = 1, 2, 3, \ldots$

(a) Diverges

(b) Converges to $-\frac{3}{7}$

(c) Converges to 1

(d) Converges to 0

(e) Converges to $\frac{3n^2+2n+1}{5-7n^2}$

Sequence converges if $\lim_{n\to\infty} a_n$ exists

$$\lim_{n\to\infty} \frac{3n^2+2n+1}{5-7n^2} = -\frac{3}{7}$$

Lead order term

7. Compute the sum of the infinite series $\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n}$.

(a) 12

(b) 9

(c) 6

(d) 4

(e) This sum diverges.

Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

$$= \sum_{n=1}^{\infty} \frac{3^2}{4}\left(\frac{3}{4}\right)^{n-1} = \frac{\frac{9}{4}}{1-3/4} = \frac{9}{4}\cdot\frac{4}{1} = 9$$

8. Calculate $\int \frac{dx}{x^3 + 2x^2}$ $= \int \frac{dx}{x^2(x+2)}$

Partial Fractions

$\frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

(a) $\frac{1}{4}\ln\left|\frac{x+2}{x}\right| - \frac{1}{2x} + C$

(b) $\frac{1}{4}\ln|x^2 + 2x| + \frac{1}{2x} + C$

(c) $\frac{1}{4}\left[(x+2)^{-2} - x^{-2}\right] - \frac{1}{2x} + C$

(d) $\frac{1}{2}\ln|x| + \arctan\left(\frac{x}{2}\right) + C$

(e) $\frac{1}{2}\ln|x| + \arctan(2x) + C$

$1 = Ax(x+2) + B(x+2) + Cx^2$

$x=0 \quad 1 = 2B \Rightarrow B = 1/2$

$x=-2 \quad 1 = 4C \quad C = 1/4$

$Ax^2 + Cx^2 = 0 \Rightarrow A = -1/4$

$\int -\frac{1}{4}\left(\frac{1}{x}\right) + \frac{1}{2}(x^{-2}) + \frac{1}{4}\left(\frac{1}{x+2}\right) dx$

$= -\frac{1}{4}\ln|x| - \frac{1}{2}(x^{-1}) + \frac{1}{4}\ln|x+2| + C = \frac{1}{4}\ln\left|\frac{x+2}{x}\right| - \frac{1}{2x} + C$

9. The nth term of a sequence is $\arctan(n)$. Which of the following statements is true?

I. The sequence diverges since $\lim_{n\to\infty} \arctan n = \infty$.

~~II. The sequence converges to 0.~~

III. The sequence converges to $\pi/2$

$\lim_{n\to\infty} a_n$ must exist

$\lim_{n\to\infty} \arctan(n) = \pi/2$

(a) Only I is true.

(b) Only II is true.

(c) Only III is true.

(d) Only I and II are true.

(e) All three statements I, II, and III, are false.

10. Which of the following statements is true of the series $\sum_{n=1}^{\infty} \frac{2n+1}{4n+7}$?

I. It converges by the Divergence Test.

II. It converges to $\frac{1}{2}$.

III. It diverges.

Divergence test $\lim_{n\to\infty} a_n$

$= \lim_{n\to\infty} \frac{2n+1}{4n+7} = 1/2 \neq 0$

so series diverges

(a) Only I is true.

~~(b) Only II is true.~~

(c) Only III is true.

(d) Only I and II are true.

(e) All three statements I, II, and III, are false.

PART II WORK OUT

Directions: Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Integrals you may find useful

$$\int \sec(\theta)\,d\theta = \ln|\sec(\theta)+\tan(\theta)|+C$$
$$\int \csc(\theta)\,d\theta = -\ln|\csc(\theta)+\cot(\theta)|+C$$

11. (10 points) Compute the arc length of $y = \frac{1}{2\pi}\ln(\cos(2\pi x))$ from $0 \le x \le \frac{1}{8}$.

$S = \int_0^{1/8} \sqrt{1+(y')^2}$ Chain Rule: $y' = \frac{1}{2\pi}\frac{1}{\cos(2\pi x)}\sin(2\pi x)(2\pi)$

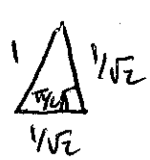
$= \tan(2\pi x)$

$$S = \int_0^{1/8}\sqrt{1+\tan^2(2\pi x)}\,dx = \int_0^{1/8}\sec(2\pi x)\,dx = \frac{1}{2\pi}\int_0^{\pi/4}\sec(u)\,du$$

$u = 2\pi x \quad du = 2\pi dx$

$x=0 \; u=0 \quad x=\frac{1}{8} \; u=\pi/4$

$$= \frac{1}{2\pi}\left[\ln|\sec(u)+\tan(u)|\right]_0^{\pi/4} = \frac{1}{2\pi}\left[\ln|\sec(\tfrac{\pi}{4})+\tan(\tfrac{\pi}{4})| - \ln|\sec(0)+\tan(0)|\right]$$



$$= \frac{1}{2\pi}\left[\ln|\sqrt{2}+1| - \ln|1+0|\right] = \frac{1}{2\pi}\ln(\sqrt{2}+1)$$

12. (10 points) Compute $\int \frac{(x+1)^2}{\sqrt{4-(x+1)^2}}\,dx$

Trigonometric Substitution

$x+1 = 2\sin\theta$

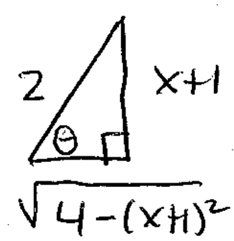
$dx = 2\cos\theta\, d\theta$

$$\Rightarrow \int \frac{(2\sin\theta)^2}{\sqrt{4-4\sin^2\theta}}\, 2\cos\theta\, d\theta = \int \frac{4\sin^2\theta}{\cancel{2\cos\theta}}\, \cancel{2\cos\theta}\, d\theta$$

$$= \int 4\sin^2\theta\, d\theta = 2\int (1-\cos 2\theta)\, d\theta = 2\left[\theta - \underbrace{\tfrac{1}{2}\sin 2\theta}\right] + C$$

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

Convert back to x



$\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left[\arcsin\left(\frac{x+1}{2}\right) - \frac{1}{\cancel{2}}(\cancel{2}\sin\theta\cos\theta)\right] + C$$

$$= 2\arcsin\left(\frac{x+1}{2}\right) - 2\left[\frac{(x+1)}{2}\,\frac{\sqrt{4-(x+1)^2}}{2}\right] + C$$

$$= \boxed{2\arcsin\left(\frac{x+1}{2}\right) - (x+1)\left(\frac{\sqrt{4-(x+1)^2}}{2}\right) + C}$$

13. (10 points) Compute $\int \frac{(4x^2 - x + 7)}{(x-1)(x^2+4)}\,dx$

Partial Fractions

$$\frac{4x^2-x+7}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4} \Rightarrow 4x^2-x+7 = A(x^2+4) + (Bx+C)(x-1)$$

$x=1$ $\quad 4(1)-1+7 = A(5)$

$10 = 5A \Rightarrow \boxed{A=2}$

$4x^2-x+7 = 2x^2+8+Bx^2+Cx-Bx-C$

$4x^2 = 2x^2+Bx^2 \Rightarrow \boxed{B=2}$

$7 = 8-C \Rightarrow \boxed{C=1}$

so we have

$$\int \frac{2}{x-1} + \frac{2x+1}{x^2+4}\,dx = \int \frac{2}{x-1} + \frac{2x}{x^2+4} + \frac{1}{4\left[\left(\frac{x}{2}\right)^2+1\right]}\,dx$$

$u_1 = x-1$ $\quad u_2 = x^2+4$ $\quad u_3 = \frac{x}{2}$

$du_1 = dx$ $\quad du_2 = 2x\,dx$ $\quad du_3 = \frac{1}{2}\,dx$

$$= 2\ln|x-1| + \ln|\underbrace{x^2+4}_{\text{always} > 0}| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$= 2\ln|x-1| + \ln(x^2+4) + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

14. (10 points) Compute the following improper integral or show that it diverges. $\int_4^{\infty} \frac{x+7}{x^2-x-6}\,dx$.

Comparison test for Integrals

Easy way: Compare to $\frac{x}{x^2} = \frac{1}{x}$

$x+7 > x$ and $x^2 - x - 6 < x^2$ so $\frac{x+7}{x^2-x-6} > \frac{x}{x^2} = 1/x$

since $\int_4^{\infty} \frac{1}{x}\,dx$ diverges $\Rightarrow \int_4^{\infty} \frac{x+7}{x^2-x-6}\,dx$ does too.

Hard way: Partial Fractions

$$\frac{x+7}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$A(x+2) + B(x-3) = x+7$

$x=-2$ $\quad -5B = 5 \Rightarrow \boxed{B=-1}$

$x=3$ $\quad 5A = 10 \Rightarrow \boxed{A=2}$

$$\int_4^{\infty} \frac{2}{x-3} + \frac{-1}{x+2}\,dx = \lim_{t\to\infty}\Big[2\ln|x-3| - \ln|x+2|\Big]_4^t$$

$u_1 = x-3$, $du_1 = dx$ $\quad u_2 = x+2$, $du_2 = dx$

$$= \lim_{t\to\infty}\Big[\ln(x-3)^2 - \ln|x+2|\Big]_4^t$$

$$= \lim_{t\to\infty}\left[\ln\left|\frac{(x-3)^2}{x+2}\right|\right]_4^t = \lim_{t\to\infty} \ln\Bigg|\underbrace{\frac{(t-3)^2}{t+2}}_{\to\infty}\Bigg| - \ln\left(\frac{1}{6}\right)$$

$= \infty - \ln(\frac{1}{6})$ diverges.

15. (5 points) Determine whether this sequence converges, and if it does, what it converges to. Clearly explain your reasoning. $\{\ln(2n+1) - \ln(3n+4)\}_{n=1}^{\infty}$

Sequence converges if $\lim_{n\to\infty} a_n$ does

$$\lim_{n\to\infty}\left[\ln(2n+1) - \ln(3n+4)\right] = \lim_{n\to\infty} \ln\left(\frac{2n+1}{3n+4}\right) = \ln\left(\frac{2}{3}\right)$$

so this converges.

16. (5 points) Determine whether this series converges, and if it does, what it converges to. Clearly explain your reasoning. $\sum_{n=1}^{\infty} \frac{1+4^n}{3^{n+1}}$ $= \sum_{n=1}^{\infty} \frac{1}{3^{n+1}} + \sum_{n=1}^{\infty} \frac{4^n}{3^{n+1}}$ (Geometric series)

$$= \sum_{n=1}^{\infty} \underbrace{\frac{1}{3^2}}_{a} \underbrace{\left(\frac{1}{3}\right)^{n-1}}_{r<1} + \sum_{n=1}^{\infty} \underbrace{\frac{4}{3^2}}_{a} \underbrace{\left(\frac{4}{3}\right)^{n-1}}_{r>1}$$

converges — Diverges since $\frac{4}{3} > 1$

so the whole thing diverges.