PRINT

LAST NAME Solutions FIRST NAME Math 152 Instructors

INSTRUCTOR: All SECTION NUMBER: All

UIN: ______ SEAT NUMBER: 0

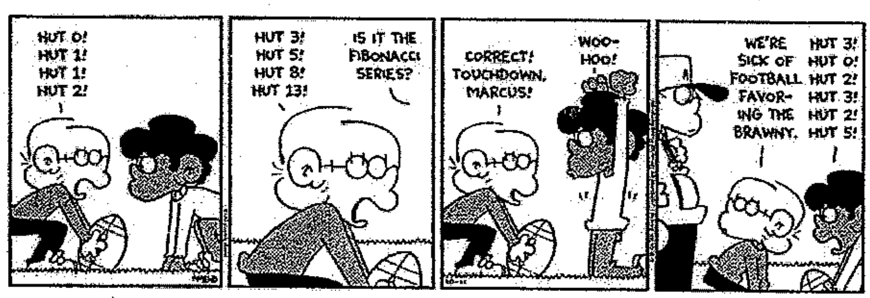
Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-10), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 11-15), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section and version letter of the exam on the Scantron form.**
5. Good Luck!

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: Instructors



http://foxtrot.com

Question	1-10	11	12	13	14-15	16	TOTAL
Points Awarded	50	10	10	10	10	10	100
Points Possible	50	10	10	10	10	10	100

1. The nth term of a sequence is $\arctan\left(-\frac{1}{n}\right)$. Which of the following statements is true?

   I. The sequence diverges since $\lim_{n\to\infty} \arctan\left(-\frac{1}{n}\right) = -\infty$.

   II. The sequence converges to 0.

   III. The sequence converges to $\pi/2$

   (a) Only I is true.

   (b) Only II is true.

   (c) Only III is true.

   (d) Only I and II are true.

   (e) All three statements I, II, and III, are false.

$\lim_{n\to\infty} -\frac{1}{n} = 0$

$\arctan(0) = 0$

so $\lim_{n\to\infty} \arctan(-1/n) = 0$

2. Which of the following integrals is equivalent to $\int_{\frac{2}{\sqrt{3}}}^{2} \frac{\sqrt{4+x^2}}{x}\,dx$?

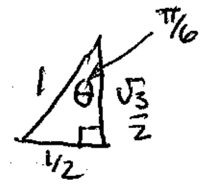
   (a) $\int_{\pi/3}^{\pi/2} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$

   (b) $\int_{\pi/6}^{\pi/4} \frac{\sec(\theta)}{\tan(\theta)}\,d\theta$

   (c) $2\int_{\pi/3}^{\pi/4} \frac{\sec(\theta)}{\tan(\theta)}\,d\theta$

   (d) $4\int_{\pi/3}^{\pi/2} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$

   (e) $2\int_{\pi/6}^{\pi/4} \frac{\sec^3(\theta)}{\tan(\theta)}\,d\theta$



Trig substitution $x = 2\tan\theta$

$x = 2 \quad 2\tan\theta = 2 \quad \tan\theta = 1 \quad \theta = \pi/4$

$x = 2/\sqrt{3} \quad 2\tan\theta = \frac{2}{\sqrt{3}} \quad \tan\theta = 1/\sqrt{3} \quad \theta = \pi/6$

$dx = 2\sec^2\theta\,d\theta$

$\int_{\pi/6}^{\pi/4} \frac{\sqrt{4+4\tan^2\theta}}{2\cancel{\sec}\tan\theta}\, 2\sec^2\theta\,d\theta = \int_{\pi/6}^{\pi/4} \frac{4\sec^3\theta}{2\tan\theta}\,d\theta$

$= 2\int_{\pi/6}^{\pi/4} \frac{\sec^3\theta}{\tan\theta}\,d\theta$

3. The integral $\int_{-1}^{1} \frac{1}{x^3}\,dx$

   (a) Converges to -2

   (b) Converges to -1

   (c) Converges to 0

   (d) Converges to 2

   (e) Diverges

$\lim_{t\to 0^-} \int_{-1}^{t} \frac{1}{x^3}\,dx + \lim_{t\to 0^+} \int_{t}^{1} \frac{1}{x^3}\,dx$

$= \lim_{t\to 0^-} \left[-\frac{x^{-2}}{2}\right]_{-1}^{t} + \lim_{t\to 0^+} \left[-\frac{x^{-2}}{2}\right]_{t}^{1}$

$= \lim_{t\to 0^-} -\frac{1}{2t^2} + \frac{1}{2} - \frac{1}{2} + \lim_{t\to 0^+} \frac{1}{2t^2} =$

$-\infty + \infty \Rightarrow$ Diverges

4. Which of the following statements is true of the series $\sum_{n=1}^{\infty} \frac{4n+3}{2n+7}$?

I. It diverges.
II. It converges to 2.
III. It converges by the Divergence Test.

(a) Only I is true.
(b) Only II is true.
(c) Only III is true.
(d) Only I and II are true.
(e) All three statements I, II, and III, are false.

Divergence test

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{4n+3}{2n+7} = 2 \neq 0$

$\Rightarrow$ Diverges

5. Which of the following integrals gives the area of the surface obtained by rotating the curve $x = e^{y/2}$ $0 \le y \le 1$ about the y-axis.

(a) $2\pi \int_0^1 y\sqrt{1+\frac{e^y}{4}}\,dy$

(b) $2\pi \int_0^1 e^{y/2}\sqrt{1+\frac{e^y}{4}}\,dy$

(c) $2\pi \int_0^2 x\sqrt{1+e^{y/2}}\,dy$

(d) $2\pi \int_0^1 y\sqrt{1+4e^y}\,dy$

(e) $2\pi \int_0^2 e^{y/2}\sqrt{1+4e^y}\,dy$

$SA = 2\pi \int_a^b r\,ds \qquad r = x = e^{y/2}$

$ds = \sqrt{1+(x')^2}\,dy \quad x' = \frac{1}{2}e^{y/2}$

$(x')^2 = \frac{1}{4}e^y$

$SA = 2\pi \int_0^1 e^{y/2}\sqrt{1+\frac{1}{4}e^y}\,dy$

6. Compute the sum of the infinite series $\sum_{n=1}^{\infty} \frac{4^{n+1}}{5^n}$.

(a) 16
(b) 12
(c) 10
(d) 8
(e) This sum diverges.

Geometric Series. Want $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

$= \sum_{n=1}^{\infty} \underbrace{\frac{4^2}{5}}_{a} \underbrace{\left(\frac{4}{5}\right)}_{r}^{n-1} = \frac{\frac{16}{5}}{1-\frac{4}{5}} = \frac{16}{5}\cdot\frac{5}{1} = 16$

7. The sequence $a_n = \frac{2n+5n^2+1}{3n^2-4n}$ for $n = 1, 2, 3, \ldots$

(a) Diverges
(b) Converges to 0
(c) Converges to 1
(d) Converges to $\frac{5}{3}$
(e) Converges to $\frac{2n+5n^2+1}{3n^2-4n}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n+5n^2+1}{3n^2-4n} = 5/3$

Lead order term

Converges

8. The partial fraction decomposition of $\frac{x+2}{(x^2-5x+6)(x-1)^2(x^2+2)}$ is

$(x^2-5x+6) = (x-3)(x-2)$

(a) $\frac{A}{x^2-5x+6}+\frac{B}{x-1}+\frac{C}{(x-1)^2}+\frac{D}{x^2+2}$

(b) $\frac{Ax+B}{x^2-5x+6}+\frac{C}{x-1}+\frac{D}{x^2+2}$

(c) $\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+2}$

(d) $\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x-1}+\frac{Dx+E}{(x-1)^2}+\frac{Fx+G}{x^2+2}$

(e) $\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x-1}+\frac{D}{(x-1)^2}+\frac{Ex+F}{x^2+2}$

$\frac{A}{x-3}+\frac{B}{x-2}+\frac{C}{x-1}+\frac{D}{(x-1)^2}+\frac{Ex+F}{x^2+2}$

9. Calculate $\int \frac{dx}{x^3-3x^2}$

Partial Fractions $\frac{1}{x^2(x-3)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$

(a) $\frac{1}{3}\ln|x| + \arctan\left(\frac{x}{3}\right)+C$

(b) $\frac{1}{3}\ln|x| + \arctan(3x)+C$

(c) $\frac{1}{9}\ln|x^2-3x| + \frac{1}{3x}+C$

(d) $\boxed{\frac{1}{9}\ln\left|\frac{x-3}{x}\right| + \frac{1}{3x}+C}$

(e) $\frac{1}{9}\left[(x-3)^{-2}+x^{-2}\right] - \frac{1}{3x}+C$

$1 = Ax(x-3)+B(x-3)+Cx^2$

$x=0 \quad 1=-3B \quad B=-1/3$

$x=3 \quad 1=9C \quad C=1/9$

$Ax^2+Cx^2=0 \Rightarrow A=-1/9$

$\int -\frac{1}{9}\left(\frac{1}{x}\right)-\frac{1}{3}(x^{-2})+\frac{1}{9}\left(\frac{1}{x-3}\right)dx$

$= -\frac{1}{9}\ln|x| + \frac{1}{3}x^{-1}+\frac{1}{9}\ln|x-3|+C = \frac{1}{9}\ln\left|\frac{x-3}{x}\right|+\frac{1}{3x}+C$

10. Compute the arc length of the curve given by the parametric equations $x=2t-13$, $y=\left(\frac{\sqrt{2}}{3}\right)t^{3/2}$, from $t=0$ to $t=10$

(a) $\frac{8}{3}(6^{3/2}-1)$

(b) $\frac{76}{3}$

(c) 152/3

(d) $\frac{4}{3}(10)^{3/2}-\frac{2}{3}$

(e) 36

$ds=\sqrt{(x')^2+(y')^2} \quad x'=2 \quad y'=\left(\frac{\sqrt{2}}{3}\right)\left(\frac{3}{2}\right)t^{1/2}$

$(x')^2=4 \quad (y')^2=\left(\frac{\sqrt{2}}{2}\right)^2 t = \frac{1}{2}t$

$\int_0^{10}\sqrt{4+\frac{1}{2}t}\,dt = 2\int_4^9 u^{1/2}du$

$u=4+\frac{1}{2}t \quad du=\frac{1}{2}dt$

$t=0 \quad u=4 \quad t=10 \quad u=9$

$=\left[2\left(\frac{2}{3}\right)u^{3/2}\right]_4^9$

$=\frac{4}{3}[27-8] = \frac{4\cdot 19}{3}=\frac{76}{3}$

$\begin{array}{r} {}^{3}19 \\ \times\ 4 \\ \hline 76 \end{array}$

PART II WORK OUT

Directions: Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

Integrals you may find useful

$$\int \sec(\theta)\,d\theta = \ln|\sec(\theta)+\tan(\theta)|+C$$
$$\int \csc(\theta)\,d\theta = -\ln|\csc(\theta)+\cot(\theta)|+C$$

11. (10 points) Compute the arc length of $y = \dfrac{1}{2\pi}\ln(\sec(2\pi x))$ from $0 \le x \le \dfrac{1}{6}$.

$ds = \sqrt{1+(y')^2}$ Use chain rule: $y' = \cancel{\frac{1}{2\pi}}\,\frac{1}{\cancel{\sec(2\pi x)}}\,\cancel{\sec(2\pi x)}\tan(2\pi x)\cancel{2\pi}$

$= \tan 2\pi x$

$$S = \int_0^{1/6}\sqrt{1+\tan^2(2\pi x)}\,dx = \int_0^{1/6}\sqrt{\sec^2(2\pi x)}\,dx = \int_0^{1/6}\sec(2\pi x)\,dx$$

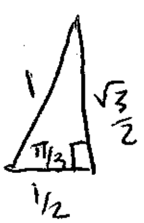
$u = 2\pi x \quad du = 2\pi\,dx$

$x=0 \quad u=0$

$x=1/6 \quad u=\frac{2\pi}{6}=\pi/3$

$$= \frac{1}{2\pi}\int_0^{\pi/3}\sec(u)\,du$$

$$= \frac{1}{2\pi}\Big[\ln|\sec(u)+\tan(u)|\Big]_0^{\pi/3}$$



$$= \frac{1}{2\pi}\left[\ln\left|\sec\tfrac{\pi}{3}+\tan\pi/3\right| - \ln|\sec(0)+\tan(0)|\right]$$

$$= \frac{1}{2\pi}\Big[\ln|2+\sqrt{3}| - \ln|1+0|\Big] = \boxed{\frac{1}{2\pi}\ln|2+\sqrt{3}|}$$

12. (10 points) Compute the following improper integral or show that it diverges. $\int_4^\infty \frac{x+5}{x^2-2x-3}\,dx$.

Easy way: compare to $\frac{x}{x^2} = 1/x$

$x+5 > x$ $\quad x^2-2x-3 < x^2$ $\quad$ so $\frac{x+5}{x^2-2x-3} > \frac{x}{x^2} = 1/x$

since $\int_4^\infty 1/x\,dx$ diverges $\Rightarrow \int_4^\infty \frac{x+5}{x^2-2x-3}\,dx$ diverges too.

Hard Way: Partial Fractions

$\frac{x+5}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$ $\quad A(x+1)+B(x-3) = x+5$

$x=-1 \quad -4B = 4 \Rightarrow B=-1$

$x=3 \quad 4A = 8 \Rightarrow A=2$

$$\int_4^\infty \frac{2}{x-3} + \frac{-1}{x+1}\,dx = \lim_{t\to\infty}\Big[2\ln|x-3| - \ln|x+1|\Big]_4^t$$

$$= \lim_{t\to\infty}\Big[\ln|x-3|^2 - \ln|x+1|\Big]_4^t = \lim_{t\to\infty}\left[\ln\frac{(x-3)^2}{|x+1|}\right]_4^t$$

$$= \lim_{t\to\infty} \ln \underbrace{\frac{(t-3)^2}{|t+1|}}_{\to\infty} - \ln\left(\tfrac{1}{5}\right) \cdot \quad \underline{\text{Diverges}}$$

13. (10 points) Compute $\int \frac{(x+2)^2}{\sqrt{9-(x+2)^2}}\,dx$

Trigonometric substitution

$x+2 = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{(3\sin\theta)^2}{\sqrt{9-9\sin^2\theta}}\, 3\cos\theta\, d\theta = \int \frac{27\sin^2\theta\cos\theta}{\sqrt{9\cos^2\theta}}\, d\theta$$

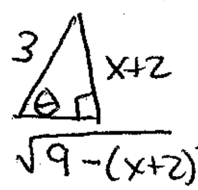
$$= \int \frac{\overset{9}{\cancel{27}}\sin^2\theta\, \cancel{\cos\theta}}{\cancel{3}\, \cancel{\cos\theta}}\, d\theta = \int 9\sin^2\theta\, d\theta$$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$= \frac{9}{2}\int 1-\cos 2\theta\, d\theta = \frac{9}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{9}{2}\left[\theta - \frac{2\sin\theta\cos\theta}{2}\right] + C = \frac{9}{2}\left[\theta + \sin\theta\cos\theta\right] + C$$

Convert back to x:



$$= \frac{9}{2}\left[\arcsin\left(\frac{x+2}{3}\right) + \frac{x+2}{3}\,\frac{\sqrt{9-(x+2)^2}}{3}\right] + C$$

$$= \boxed{\frac{9}{2}\arcsin\left(\frac{x+2}{3}\right) + \frac{(x+2)\sqrt{9-(x+2)^2}}{2} + C}$$

14. (5 points) Determine whether this series converges, and if it does, what it converges to. Clearly explain your reasoning. $\sum_{n=1}^{\infty} \frac{1+3^n}{2^{n+1}}$ $= \sum_{n=1}^{\infty} \frac{1}{2^{n+1}} + \sum_{n=1}^{\infty} \frac{3^n}{2^{n+1}}$

$= \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{\infty} \frac{3}{4}\left(\frac{3}{2}\right)^{n-1}$ Geometric Series

$r = 1/2$ converges

$r = 3/2 \Rightarrow$ Diverges

Since one of the geometric series diverges they both diverge.

15. (5 points) Determine whether this sequence converges, and if it does, what it converges to. Clearly explain your reasoning. $\{\ln(1+3n) - \ln(2n+4)\}_{n=1}^{\infty}$

Sequence converges if $\lim_{n \to \infty} a_n$ exists

$\lim_{n \to \infty} \left[\ln(1+3n) - \ln(2n+4)\right] = \lim_{n \to \infty} \ln\left(\frac{1+3n}{2n+4}\right) =$

$\ln(3/2)$

<u>converges</u>

16. (10 points) Compute $\int \frac{(3x^2-3x+10)}{(x-2)(x^2+4)}\,dx$

Partial Fractions $\frac{3x^2-3x+10}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4}$

$3x^2-3x+10 = A(x^2+4) + (Bx+C)(x-2)$

$x=2 \quad 3(2)^2-3(2)+10 = A(2^2+4) \Rightarrow 16 = 8A$ or $\boxed{A=2}$

$3x^2-3x+10 = 2x^2+8 + Bx^2 + Cx - 2Bx - 2C$

$3x^2 = 2x^2 + Bx^2 \Rightarrow \boxed{B=1}$

$10 = 8-2C \Rightarrow \boxed{C=-1}$

$\int \frac{2}{x-2} + \frac{x-1}{x^2+4}\,dx = 2\ln|x-2| + \int \frac{x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx$

$u = x^2+4$

$du = 2x\,dx$

$= 2\ln|x-2| + \frac{1}{2}\ln\underbrace{|x^2+4|}_{\text{always} > 0} - \frac{1}{4}\int \frac{1}{(\frac{x}{2})^2+1}\,dx$

$u = x/2 \quad du = \frac{1}{2}dx$

$= \boxed{2\ln|x-2| + \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$