PRINT

LAST NAME Vargo FIRST NAME James

INSTRUCTOR: __________ SECTION NUMBER: __________

UIN: __________ SEAT NUMBER: __________

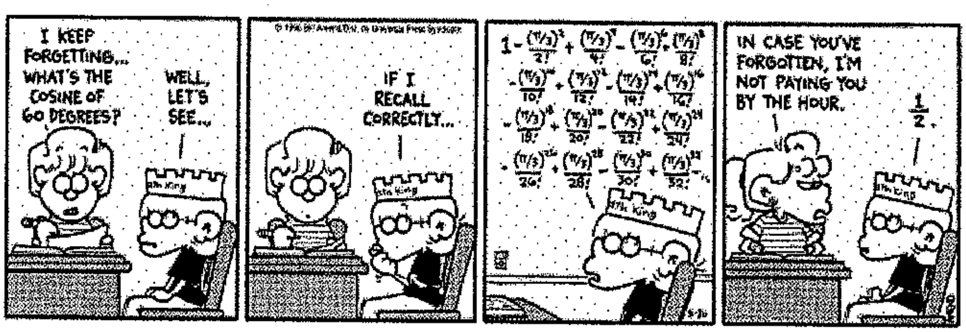
Directions

1. The use of all electronic devices is prohibited.
2. In Part 1 (Problems 1-12), mark the correct choice on your Scantron using a No. 2 pencil. **Record your choices on your exam. Scantrons will not be returned.**
3. In Part 2 (Problems 13-17), present your solutions in the space provided. **Show all your work neatly and concisely and clearly indicate your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section and version letter of the exam on the Scantron form.**
5. Any scratch paper used must be handed in with the exam.
6. Good Luck!

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ____________________



http://foxtrot.com

Question	1-12	13	14	15	16	17	TOTAL
Points Awarded							
Points Possible	48	10	14	7	11	10	100

1. Find a unit vector in the direction of the vector $\langle\sqrt{3}, -3, 2\rangle$.

   (a) $4\langle\sqrt{3}, -3, 2\rangle$

   (b) $\frac{1}{16}\langle\sqrt{3}, -3, 2\rangle$

   (c) $16\langle\sqrt{3}, -3, 2\rangle$

   (d) $\frac{1}{16}\langle-\sqrt{3}, 3, -2\rangle$

   (e) $\frac{1}{4}\langle\sqrt{3}, -3, 2\rangle$

2. What is the distance between points $(-2, -1, 4)$ and $(1, -3, -2)$?

   (a) 3

   (b) $\sqrt{29}$

   (c) 7

   (d) $\sqrt{61}$

   (e) 49

3. What is the Maclaurin series for $e^{\frac{x^2}{2}}$?

   (a) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{2(n!)}$

   (b) $\sum_{n=0}^{\infty} \frac{x^{n}}{2(n!)}$

   (c) $\sum_{n=0}^{\infty} \frac{x^{n}}{2^n(n!)}$

   (d) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n(n!)}$

   (e) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n(2n!)}$

4. Which series diverges?

(a) $\sum_{n=2}^{\infty} \frac{n^2-2n-1}{n^3+4n}$

(b) $\sum_{n=0}^{\infty} \frac{1}{n^2+2n+4}$

(c) $\sum_{n=1}^{\infty} ne^{-n^2}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

5. Which series converges absolutely?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{e^{n+1}}$

(c) $\sum_{n=1}^{\infty} (\sqrt{n+2}-\sqrt{n})$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(e) $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

6. What is the power series representation of $f(x) = \frac{1}{9-4x^2}$ at $x = 0$?

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{2n}}{9^{n+1}}$

(b) $\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{9^{n+1}}$

(c) $\sum_{n=0}^{\infty} \frac{9^n x^{2n}}{4^{n+1}}$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{9^n x^{2n}}{4^{n+1}}$

(e) $\sum_{n=1}^{\infty} \frac{4^n x^{2n}}{9}$

7. Given that the power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 3$ and diverges when $x = 6$, which of the statements is certain to be true?

   (a) $\sum_{n=0}^{\infty} c_n(-6)^n$ is divergent

   (b) $\sum_{n=0}^{\infty} c_n(-4)^n$ is convergent

   (c) $\sum_{n=0}^{\infty} c_n(-3)^n$ is convergent

   (d) $\sum_{n=0}^{\infty} c_n(-2)^n$ is convergent

   (e) None of these statements is certain to be true.

8. What is the cosine of the angle between the vectors $\langle -1, 2, -3\rangle$ and $\langle 3, -2, -1\rangle$?

   (a) $\frac{-5}{\sqrt{7}}$

   (b) $\frac{5}{\sqrt{7}}$

   (c) $\frac{-5}{7}$

   (d) $\frac{-2}{7}$

   (e) $\frac{2}{\sqrt{7}}$

9. Let $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ be nonzero vectors where $\|\mathbf{a}\|$ is the length of $\mathbf{a}$. Given

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\|$$

and

$$0 < |\mathbf{c} \cdot \mathbf{d}| < \|\mathbf{c}\| \|\mathbf{d}\|$$

Which of these statements is true?

   (a) $\mathbf{a}$ and $\mathbf{b}$ are parallel; $\mathbf{c}$ and $\mathbf{d}$ are neither orthogonal nor parallel

   (b) $\mathbf{a}$ and $\mathbf{b}$ are orthogonal; $\mathbf{c}$ and $\mathbf{d}$ are neither orthogonal nor parallel

   (c) $\mathbf{a}$ and $\mathbf{b}$ are neither orthogonal nor parallel; $\mathbf{c}$ and $\mathbf{d}$ are orthogonal.

   (d) $\mathbf{a}$ and $\mathbf{b}$ are neither orthogonal nor parallel; $\mathbf{c}$ and $\mathbf{d}$ are parallel.

   (e) None of the above statements is true.

10. Find the vector projection of the vector $\langle 3, 2, 1\rangle$ onto the vector $\langle 1, 2, 3\rangle$.

(a) $\frac{5}{7}$

(b) $\frac{10}{\sqrt{14}}$

(c) $\frac{1}{14}\langle 30, 20, 10\rangle$

(d) $\frac{1}{14}\langle 10, 20, 30\rangle$

(e) $\frac{1}{\sqrt{14}}\langle 30, 20, 10\rangle$

11. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n(x-5)^n}{n!}$.

(a) 0

(b) $\frac{1}{3}$

(c) 3

(d) $\frac{16}{3}$

(e) ∞

12. The equation of the sphere passing through the point $(1, 2, 3)$ with center $(2, -1, 3)$ is

(a) $(x-1)^2 + (y-2)^2 + (z-3)^2 = 10$

(b) $(x-2)^2 + (y+1)^2 + (z-3)^2 = 10$

(c) $(x-1)^2 + (y-2)^2 + (z-3)^2 = \sqrt{10}$

(d) $(x-2)^2 + (y+1)^2 + (z-3)^2 = \sqrt{10}$

(e) $(x-1)^2 + (y-2)^2 + (z-3)^2 = 100$

PART II WORK OUT

Directions: Present your solutions in the space provided. **Show all your work neatly and concisely and box your final answer.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

13. (10 points) Find the radius and center of the sphere

$$x^2 + y^2 + z^2 - 4x + 2y - 6z = 11$$

Complete the square:

$$(x^2 - 4x + 4 - 4) + (y^2 + 2y + 1 - 1) + (z^2 - 6z + 9 - 9) = 11$$

$$(x-2)^2 - 4 + (y+1)^2 - 1 + (z-3)^2 - 9 = 11$$

$$(x-2)^2 + (y+1)^2 + (z-3)^2 = 25$$

$$\boxed{\text{Center } (2, -1, 3) \qquad \text{Radius } 5}$$

14. (8 points) Find the first four terms of the Taylor series for $f(x) = x^{\frac{3}{2}}$ centered at $a = 4$.

k	$f^{(k)}(x)$	$f^{(k)}(a=4)$
0	$x^{3/2}$	8
1	$\frac{3}{2}x^{1/2}$	3
2	$\frac{3}{4}x^{-1/2}$	$\frac{3}{8}$
3	$\frac{-3}{8}x^{-3/2}$	$-\frac{3}{64}$

$$T_3(x) = \sum_{k=0}^{3} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$\boxed{= 8 + 3(x-4) + \frac{3}{8 \cdot 2!}(x-4)^2 - \frac{3}{64 \cdot 3!}(x-4)^3}$$

(6 points) Use Taylor's Inequality to give a bound for the error for when using $T_1(x)$ (the first degree Taylor polynomial) centered at $a = 4$ to approximate $f(x) = x^{\frac{3}{2}}$ on $[3, 5]$.

Taylor's Inequality: $|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$ on the interval $[3, 5]$ where $M = \max|f^{(n+1)}(x)|$ on $[3, 5]$.

$$|R_1(x)| \le \frac{M}{2!}|x-4|^2 \qquad M = \max_{x \in [3,5]} |f^{(2)}(x)|$$

$$|f''(x)| = \left|\frac{3}{4}\frac{1}{\sqrt{x}}\right| \quad \text{attains its maximum @ } x = 3$$

$$\le \frac{3}{4\sqrt{3}} = \frac{\sqrt{3}}{4} = M$$

$$|R_1(x)| \le \boxed{\frac{\sqrt{3}}{4 \cdot 2!}|x-4|^2 \le \frac{\sqrt{3}}{8}}$$

15. (7 points) Compute the Maclaurin series for $\dfrac{\cos(x)-1}{x^2}$.

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\cos(x) - 1 = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\frac{\cos(x)-1}{x^2} = -\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \ldots$$

$$\boxed{= \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n+2)!}}$$

16. (6 points) The series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{(3^n)(2n^2-n)}$ converges conditionally. Determine whether this series also converges absolutely. Clearly explain your reasoning.

Ratio Test

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(-1)^{n+1}(n+1)}{3^{n+1}\left[2(n+1)^2-(n+1)\right]} \Big/ \frac{(-1)^n n}{3^n(2n^2-n)} \right|$$

$$= \left| \frac{(-1)^{n+1}}{(-1)^n} \frac{(n+1)}{n} \frac{3^n}{3^{n+1}} \frac{(2n^2-n)}{(2n^2+4n+2-n-1)} \right|$$

$$= \frac{n+1}{n} \cdot \frac{(2n^2-n)}{(2n^2+3n+1)} \cdot \frac{1}{3} \xrightarrow{\text{as } n\to\infty} 1 \cdot 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3} < 1$ So it converges absolutely.

(5 points) What is a bound on the error if we sum the first 3 terms of the series?

The series is alternating, so

$$|\text{error}| \le |a_4| = \left| \frac{(-1)^4 \cdot 4}{3^4(2\cdot 4^2-4)} \right| = \frac{4}{81(\cdot 28)}$$

$$= \frac{1}{81\times 7} = \frac{1}{567}$$

17. (10 points) Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-5)^n}{3^n(n^2+2)}$$

Ratio test

$$\lim_{n\to\infty} \left| \frac{(x-5)^{n+1}}{3^{n+1}((n+1)^2+2)} \Big/ \frac{(x-5)^n}{3^n(n^2+2)} \right|$$

$$= \lim_{n\to\infty} \frac{|x-5|}{3} \cdot \frac{n^2+2}{n^2+2n+3} = \frac{|x-5|}{3}$$

$$< 1 \quad \text{if} \quad |x-5| < 3.$$

Radius of Convergence = 3

Center of interval = 5. Endpoints: $5-3=2$

$5+3=8$

Check Endpoints

@ $x=2$, $\sum_{n=0}^{\infty} \frac{(-3)^n}{3^n(n^2+2)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+2}$

Converges by Alternating Series Test.

@ $x=8$, $\sum_{n=0}^{\infty} \frac{3^n}{3^n(n^2+2)} = \sum_{n=0}^{\infty} \frac{1}{n^2+2}$

$\frac{1}{n^2+2} < \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series with $p=2>1$)

So by the Comparison Test, $\sum_{n=0}^{\infty} \frac{1}{n^2+2}$ converges.

$\boxed{\text{Answer} \quad [2, 8]}$