

MATH 152

Spring 1995

Exam III

Test Form B

NAME

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (Problems 1—8), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.
2. In Part II (Problems 9—15), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. **CLEARLY INDICATE YOUR FINAL ANSWER.**

MATH 152, Spring 1995, Exam III, Test Form B

Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour. (6 points each)

1. The parametric equations $x = \cos 4\pi t$, $y = \sin 4\pi t$ give the position of a particle on the unit circle at time t . As t goes from 0 to 1, the particle goes around the circle:

- (a) once counter-clockwise. (b) once clockwise. (c) 4π times counter-clockwise.
(d) twice clockwise. (e) twice counter-clockwise.

2. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n^2}$ is

- (a) Divergent by oscillation. (b) Conditionally, but not absolutely, convergent.
(c) Divergent. (d) Conditionally, but not absolutely, divergent.
(e) Absolutely convergent.

3. Given that the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$ is 6, what is the radius of convergence of $\sum_{n=1}^{\infty} 2^n a_n x^n$?

- (a) 1 (b) 2 (c) 3 (d) 6 (e) 12

4. Given that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ (for $|x| < 1$), then (for $|x| < 1$) we have $\sum_{n=0}^{\infty} n x^n =$

- (a) $\frac{1}{1-x}$ (b) $\frac{1}{(1-x)^2}$ (c) $\frac{x}{1-x}$ (d) $\frac{x}{(1-x)^2}$ (e) $\frac{n}{1-x}$

5. The radius of convergence of $\sum_{n=1}^{\infty} \frac{3^n (x-1)^n}{n^2}$ is

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) 3

6. The series $\sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}$ is

- (a) Divergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$. (b) Convergent by the ratio test.
(c) Divergent by the root test. (d) Convergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
(e) Divergent by the ratio test.

7. Given that $P = \sum_{n=1}^{10} \frac{1}{n^3} = 1.197531986$, which of the calculations below gives the best (closest) estimate for $S = \sum_{n=1}^{\infty} \frac{1}{n^3}$? (Assume all arithmetic reported is correct.)

- (a) Carry on by adding $\sum_{n=11}^{20} \frac{1}{n^3} = .0033358563$ to get $S \approx 1.200867842$.
(b) Use $S \approx \int_1^{\infty} \frac{1}{t^3} dt = 0.500$.
(c) Use $S \approx P + \frac{1}{2} \left(\int_{10}^{\infty} \frac{1}{t^3} dt + \int_{11}^{\infty} \frac{1}{t^3} dt \right) = 1.2020981014$.
(d) Use $S \approx P + \int_{10}^{\infty} \frac{1}{t^3} dt = 1.202531986$.
(e) Execute the Maple code:

```
P:=1.197531986; q:=0;
for n from 10 to 10000 do    q := q + 1/n^3;    od;
P + q;
```

8. The series $\sum_{n=0}^{\infty} \frac{1}{2^n n!}$ converges to

- (a) $\ln 2$ (b) \sqrt{e} (c) $\sinh(1/2)$ (d) $\sin(2)$ (e) e^2

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected. Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

9. (7 points) Using the fact that $\ln(1+t) = t - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \dots$,

(a) Find the 4th degree Taylor polynomial approximation at $c = 0$ to $\ln(1+x^2)$.

(b) Using the result from (a), estimate $\int_0^{1/2} \ln(1+x^2) dx$.

10. (8 points) Consider the curve given by the parametric equations $x = \frac{1}{3}t^3 + t$, $y = 3t^3 - t$, for $t \geq 0$. For what value of t is the slope of the tangent line to this curve equal to -1 ?

11. (8 points) Find the arc length of the path traced by the parametric equations $x = \cos^2 t$, $y = \sin^2 t$ as t goes from 0 to $\frac{\pi}{2}$.

12. (7 points) Let $H(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n!)^2}$.

What is the minimum number of terms needed to evaluate $H(1)$ to within $\pm 10^{-10}$?

13. (8 points) Find the 3rd degree Taylor polynomial $T_3(x)$ for $f(x) = \frac{1}{x}$ at the number $c = 2$.

14. (6 points) Find the area between the parametric curve $x = t + t^3$, $y = t - t^3$ and the x -axis as shown below in the Maple plot generated by the command:

```
plot([t + t^3, t - t^3, t=0..1]);
```

15. (8 points) Let $f(x) = x^2 \cos x$.

(a) Find the Maclaurin series for $f(x)$, including the general term.

(b) Find $f^{(6)}(0)$.