

Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour.

(5 points each)

1. In the Maclaurin series for $\frac{\sin(x)}{x}$, what is the coefficient of x^4 ?

- (a) $-\frac{1}{4!}$ (b) $-\frac{1}{5!}$ (c) 0 (d) $\frac{1}{5!}$ (e) $\frac{1}{4!}$

2. Find the center, c , and radius of convergence, r , of the series $\sum_{n=1}^{\infty} \frac{n(x+2)^n}{5^{n-1}}$.

- (a) $c = 2, r = 5$ (b) $c = 2, r = \frac{1}{5}$ (c) $c = -2, r = 5$
(d) $c = -2, r = \frac{1}{5}$ (e) $c = -\frac{2}{5}, r = 1$

3. The series $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots$ converges to e .

So the series $\frac{3}{1!} + \frac{3}{2!} + \frac{3}{3!} + \frac{3}{4!} + \dots + \frac{3}{(n+1)!} + \dots$ must converge to the limit

- (a) $3e - 3$ (b) $3e - 1$ (c) $3e$ (d) $3e + 1$ (e) $3e + 3$

4. Let $f(x)$ be the function whose Taylor series at 3 is $f(x) = \sum_{n=1}^{\infty} \frac{1 + \sqrt{n}}{(n+1)!} (x-3)^n$.

Find $f^{(16)}(3)$, i.e., find the 16-th derivative of f at 3.

- (a) $\frac{5}{17!}$ (b) $\frac{5}{16!}$ (c) $\frac{5}{17}$ (d) 5 (e) $5(3)^{16}$

5. The series $\sum_{n=1}^{\infty} (-1)^n \frac{3n}{n^2 + 1}$ is

(a) absolutely convergent.

(b) conditionally convergent.

(c) divergent.

(d) conditionally divergent.

6. The series $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ is

(a) convergent by the integral test.

(b) convergent by the ratio test.

(c) convergent by the comparison test.

(d) convergent by the limit comparison test.

(e) divergent.

7. Find the radius of convergence of the Maclaurin series for e^{-x} .

(a) $-\infty$

(b) 0

(c) 1

(d) e

(e) ∞

8. Find an equation of the tangent line to the parametric curve $x = 3t^2 - 2t$, $y = 2t^3 + t$ at the point where $t = 2$.

(a) $y = \frac{2}{5}x$

(b) $y = (6t^2 + 1)(x - 8) + 18$

(c) $y = \frac{2}{5}x + \frac{74}{5}$

(d) $y = \frac{5}{2}x - 2$

(e) $y = 25x - 152$

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

12. (9 points)

(a) Find the total length of the astroid $x = \cos^3(\theta)$, $y = \sin^3(\theta)$.

(b) Find the surface area obtained by rotating the astroid about the x -axis.

13. (5 points) Find parametric equations for the line segment from the point $(2, 3)$ to the point $(2, 5)$. Be sure to give the domain of the parameter.

14. (8 points)

(a) Find the Taylor polynomial of degree 2 for the function $f(x) = \ln(x)$ at $c = 3$.

(b) Give the remainder term.

(c) Use the remainder term to estimate the maximum possible error when the polynomial is used to approximate $\ln(x)$ on the interval $[2, 4]$.

15. (5 points) Determine if the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ converges or diverges.

16. (5 points) Determine if the series $\sum_{n=1}^{\infty} \frac{n^2+1}{2^n}$ converges or diverges.

17. (5 points) Determine if the series $\sum_{n=1}^{\infty} n e^{-n^2}$ converges or diverges.

18. (8 points)

- (a) Find the Maclaurin series of the function $f(x) = \sin(x^2)$. Give at least 4 non-zero terms or the general term.

- (b) Use this series for $\sin(x^2)$ to approximate $\int_0^{0.1} \sin(x^2) dx$ to within $\pm 10^{-10}$. Justify the number of terms you use.