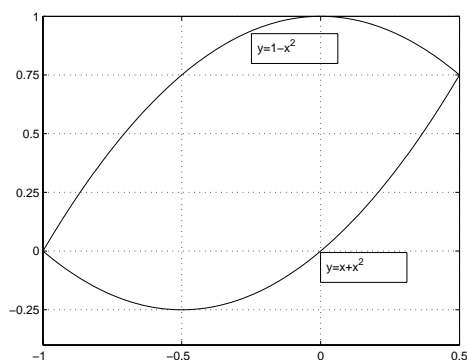


Math 152  
Common Exam #1 Solutions  
Fall 2005

1. The correct answer is (c).

$$\text{Area} = \int_{-1}^{\frac{1}{2}} [(1-x^2) - (x+x^2)] dx = \int_{-1}^{\frac{1}{2}} (1-2x^2-x) dx$$



2. The correct answer is (d).

The average value of  $f(x) = 3x^2 - 2x$  on the interval  $[-1, 1]$  is

$$f_{\text{ave}} = \frac{1}{1 - (-1)} \int_{-1}^1 (3x^2 - 2x) dx = \frac{1}{2} (x^3 - x^2) \Big|_{-1}^1 = 1$$

and

$$\begin{aligned} f(c) &= f_{\text{ave}} \\ 3c^2 - 2c &= 1 \\ 3c^2 - 2c - 1 &= 0 \\ (3c+1)(c-1) &= 0 \\ c &= -\frac{1}{3}, 1 \end{aligned}$$

3. The correct answer is (b).

$$\begin{aligned}\int \sin^2 x \cos^2 x \, dx &= \int (\sin x \cos x)^2 \, dx \\ &= \int \frac{1}{4} \sin^2 2x \, dx \\ &= \frac{1}{8} \int (1 - \cos 4x) \, dx \\ &= \frac{1}{8} \left( x - \frac{1}{4} \sin 4x + C \right) \\ &= \frac{1}{8} x - \frac{1}{32} \sin 4x + C\end{aligned}$$

4. The correct answer is (b).

Let  $u = x + 1$ . Then

$$\begin{aligned}\int_0^3 \frac{x}{\sqrt{x+1}} \, dx &= \int_1^4 \frac{u-1}{\sqrt{u}} \, du \\ &= \int_1^4 (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) \, du \\ &= \left( \frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right) \Big|_1^4 \\ &= \left( \frac{2}{3}(8) - 4 \right) - \left( \frac{2}{3} - 2 \right) \\ &= \frac{8}{3}\end{aligned}$$

5. The correct answer is (a).

Let  $u = \ln x$  and  $v = \frac{1}{2}x^2$ . Then

$$\begin{aligned}\int_1^2 x \ln x \, dx &= \frac{1}{2} x^2 \ln x \Big|_1^2 - \int_1^2 \frac{1}{2} x \, dx \\ &= 2 \ln 2 - \frac{x^2}{4} \Big|_1^2 \\ &= 2 \ln 2 - \frac{3}{4}\end{aligned}$$

6. The correct answer is (d).

$$f'(x) = \left( \sqrt{\frac{1}{x^3} + 1} \right) \times \frac{d}{dx} \left( \frac{1}{x} \right) = -\frac{1}{x^2} \sqrt{\frac{1}{x^3} + 1}$$

7. The correct answer is (c).

8. The correct answer is (c).

Using partial fractions:

$$\begin{aligned} \frac{1}{x(x-1)(x-2)} &= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2} \\ &= \frac{A(x-1)(x-2)}{x(x-1)(x-2)} + \frac{Bx(x-2)}{x(x-1)(x-2)} + \frac{Cx(x-1)}{x(x-1)(x-2)} \\ &= \frac{(A+B+C)x^2 + (-3A-2B-C)x + 2A}{x(x-1)(x-2)}, \end{aligned}$$

giving

$$\begin{aligned} A + B + C &= 0 \\ -3A - 2B - C &= 0 \\ 2A &= 1 \end{aligned}$$

and  $A = 1/2$ ,  $B = -1$  and  $C = 1/2$ . Then

$$\int \frac{1}{x(x-1)(x-2)} dx = \int \left( \frac{1}{2x} - \frac{1}{x-1} + \frac{1}{2(x-2)} \right) dx = \frac{1}{2} \ln|x| - \ln|x-1| + \frac{1}{2} \ln|x-2| + C$$

9. The correct answer is (b).

$$W = \int_0^1 3 \sin \pi x dx = -\frac{3}{\pi} \cos \pi x \Big|_0^1 = -\frac{3}{\pi} (\cos \pi - \cos 0) = -\frac{3}{\pi} (-1 - 1) = \frac{6}{\pi}$$

10. The correct answer is (c).

Let  $x = \sqrt{3} \tan u$ . Then

$$\begin{aligned}\int \frac{x}{x^2 + 3} dx &= \int \frac{(\sqrt{3} \tan u) \sqrt{3} \sec^2 u}{3(\tan^2 u + 1)} du \\ &= \int \tan u \, du \\ &= -\ln |\cos u| + C \\ &= -\ln \sqrt{3/(x^2 + 3)} + C \\ &= \frac{1}{2} \ln((x^2 + 3)/3) + C\end{aligned}$$

and

$$\begin{aligned}\int_2^4 \frac{x}{x^2 + 3} dx &= \left. \frac{1}{2} \ln((x^2 + 3)/3) \right|_2^4 \\ &= \frac{1}{2} \ln(19/3) - \frac{1}{2} \ln(7/3) \\ &= \frac{1}{2} (\ln 19 - \ln 3) - \frac{1}{2} (\ln 7 - \ln 3) \\ &= \frac{1}{2} \ln 19 - \frac{1}{2} \ln 7\end{aligned}$$

11. (a) Let  $x + 1 = \tan u$ . Then

$$\begin{aligned}\int \frac{dx}{x^2 + 2x + 2} &= \int \frac{dx}{(x + 1)^2 + 1} \\ &= \int \frac{\sec^2 u}{\tan^2 u + 1} du \\ &= \int 1 \, du \\ &= u + C \\ &= \tan^{-1}(x + 1) + C\end{aligned}$$

and

$$\int_0^1 \frac{dx}{x^2 + 2x + 2} = \tan^{-1}(x + 1) \Big|_0^1 = \tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1} 2 - \frac{\pi}{4}$$

(b)

$$\begin{aligned}\int \tan^3 x \sec x \, dx &= \int \tan^2 x \sec x \tan x \, dx \\ &= \int (\sec^2 x - 1) \sec x \tan x \, dx \\ &= \int \sec^2 x (\sec x \tan x) \, dx - \int \sec x \tan x \, dx \\ &= \frac{\sec^3 x}{3} - \sec x + C\end{aligned}$$

(c) Let  $u = x^2$  and  $v = \frac{1}{2}e^{2x}$ . Then

$$\int x^2 e^{2x} \, dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} \, dx$$

Now let  $u = x$  and  $v = \frac{1}{2}e^{2x}$ . Then

$$\int x e^{2x} \, dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} \, dx = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$$

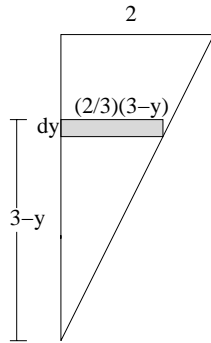
and

$$\int x^2 e^{2x} \, dx = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x}$$

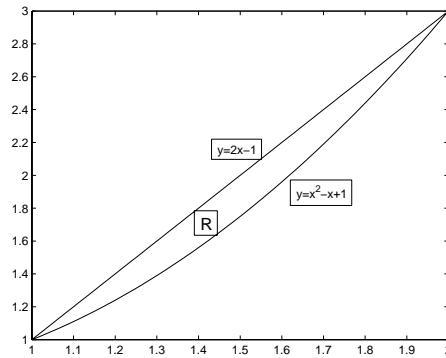
(d) Let  $x = 2 \sin u$ . Then

$$\begin{aligned}\int \sqrt{4 - x^2} \, dx &= \int \sqrt{4 - 4 \sin^2 u} (2 \cos u) \, du \\ &= 4 \int \cos^2 u \, du \\ &= 2 \int (1 + \cos 2u) \, du \\ &= 2u + \sin 2u + C \\ &= 2u + 2 \sin u \cos u + C \\ &= 2u + 2 \sin u \sqrt{1 - \sin^2 u} + C \\ &= 2 \sin^{-1}(x/2) + x \sqrt{1 - (x^2/4)} + C\end{aligned}$$

$$12. W = \int_0^3 (62.5)(8)(2/3)(3-y)y dy = 1500 \text{ N}$$



13. The region R enclosed is shown below.



The volume of the solid obtained by revolving R about the  $y$ -axis is

$$\begin{aligned} V &= \int_1^2 2\pi x[(2x-1) - (x^2-x+1)] dx \\ &= 2\pi \int_1^2 x(-x^2+3x-2) dx \\ &= 2\pi \int_1^2 (-x^3+3x^2-2x) dx \\ &= 2\pi \left( -\frac{x^4}{4} + x^3 - x^2 \right) \Big|_1^2 \\ &= \frac{\pi}{2} \end{aligned}$$

14. The equilateral triangle resulting from the intersection of the solid with the plane  $P_x$  has side of length  $s_x = 2\sqrt{9 - x^2}$  and area  $A(x) = (s_x)^2\sqrt{3}/4 = \sqrt{3}(9 - x^2)$ . The volume of the solid is

$$V = \int_{-3}^3 A(x) dx = \int_{-3}^3 \sqrt{3}(9 - x^2) dx = \sqrt{3} \left( 9x - \frac{x^3}{3} \right) \Big|_{-3}^3 = 36\sqrt{3}$$