Fall 2002 **Math 152** **Exam 3** **Test Form A**

PRINT Surname:____________________ **Rest of name:**____________________

Signature:____________________ **Student ID:**____________________

Instructor:____________________ **Section #:**____________________

Calculators may be used only during the last 30 minutes of the test. The ScanTron forms will be collected at the end of the test. **Calculators may not be used to perform "calculus" operations, such as attempting to find sums of infinite series!** Aggies do not **lie**, **cheat**, or steal, nor tolerate those who do.

In Part I, mark the correct choice on your ScanTron with a #2 pencil. For your own records, also mark your choices on your test paper, because your ScanTron will not be returned. **Do not use the ScanTron as scratch paper.** Remember to write your name, section, and test form (A or B) on the ScanTron!

In Part II, all work to be graded must be shown in the space provided, or clearly pointed to therefrom, and your final answer must be clearly indicated. You may use the back of any page for scratch work; any other paper used should be turned in with the test.

POSSIBLY USEFUL INFORMATION

$$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1},$$

where M is an upper bound on $|f^{(n+1)}(c)|$ for all c in the interval concerned.

x	$\tan^{-1} x$
$-\infty$	$-\pi/2$
-1	$-\pi/4$
0	0
1	$\pi/4$
∞	$\pi/2$

Part I: Multiple Choice (4 points each)

There is no partial credit. Do not use a calculator to find limits or to sum infinite series.

1. A series $\sum_{n=1}^{\infty} a_n$ cannot converge unless

   (A) $|a_n|$ is a decreasing sequence.

   (B) $a_n \leq \frac{1}{n^p}$ with $p > 1$.

   (C) $\lim_{n\to\infty} a_n = 0$.

   (D) $\sum_{n=1}^{\infty} |a_n|$ converges.

   (E) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$.

2. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{7^{n+1}} =$

   (A) $\frac{2}{7}$

   (B) $\frac{1}{35}$

   (C) $\frac{9}{7}$

   (D) $\frac{7}{2}$

   (E) $\frac{1}{9}$

3. If point A has coordinates $(3,2,1)$ and point B has coordinates $(0,-1,2)$, then B is displaced from A by the vector $\overrightarrow{AB} =$

(A) $\langle 3,-3,2\rangle$

(B) $\langle 3,1,3\rangle$

(C) $\langle -3,3,2\rangle$

(D) $\langle 3,3,1\rangle$

(E) $\langle -3,-3,1\rangle$

4. The surface having the equation $x^2+y^2+z^2+4x-2z=20$ is

(A) a sphere with center at $(-2,0,1)$ and radius 5.

(B) a sphere with center at $(4,0,-2)$ and radius 4.

(C) a sphere with center at $(2,0,-1)$ and radius $\sqrt{20}$.

(D) a sphere with center at $(-4,0,2)$ and radius $\sqrt{20}$.

(E) not a sphere at all.

5. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ do you need to add to find the sum to within an error of 10^{-12}?

(A) 10

(B) 100

(C) 10^4

(D) 10^{12}

(E) 10^{48}

6. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ represents

(A) $\dfrac{1}{(1-x)^2}$

(B) $\sin x$

(C) $\ln(x-1)$

(D) e^x

(E) $\cos x$

7. $\displaystyle\lim_{n\to\infty} \frac{n^2-2n+1}{2n^2+3} =$

(A) $\dfrac{1}{3}$

(B) 0

(C) ∞

(D) $\dfrac{1}{2}$

(E) 1

8. $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{n+\sin(n\pi/3)} =$

(A) ∞

(B) $\dfrac{1}{2}$

(C) 1

(D) 0

(E) [does not exist]

9. The first three terms of the Taylor series of $f(x) = \dfrac{1}{\sqrt{x}}$ around (i.e., centered at) $x = 4$ are

(A) $x^{-1/2} - \dfrac{1}{2}x^{-3/2}(x-4) + \dfrac{3}{4}x^{-5/2}(x-4)^2$

(B) $\dfrac{1}{2} - \dfrac{1}{16}(x-4) + \dfrac{3}{256}(x-4)^2$

(C) $4^{-1/2} - \dfrac{1}{2}4^{-3/2} + \dfrac{3}{8}4^{-5/2}$

(D) $1 - \dfrac{1}{2}(x-4) + \dfrac{3}{4}(x-4)^2$

(E) $1 - \dfrac{1}{2}x^{-3/2}4 + \dfrac{3}{8}x^{-5/2}4^2$

10. The remainder estimate for Taylor series gives us this upper bound on the error of the Taylor approximation in Question 9 for all x in the interval $2 \le x \le 6$:

(A) $\frac{5\sqrt{2}}{32}$

(B) $\frac{7\sqrt{2}}{16}$

(C) $\frac{3\sqrt{2}}{64}$

(D) $\frac{7\sqrt{2}}{4!}$

(E) $\frac{4\sqrt{2}}{3!}$

11. Which of these series is convergent but not absolutely convergent?

(A) $\sum_{n=1}^{\infty} n^{1/10} \cos(n\pi)$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{\sqrt{n}}}$

(E) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

12. The series $\sum_{n=1}^{\infty} \frac{n^2}{n^3 - \frac{1}{2}}$

(A) diverges, by the ratio test.

(B) converges, by the ratio test.

(C) diverges, by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) converges, by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^3}$.

(E) converges, by comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$.

Part II: Write Out (10 points each except as indicated)

Give complete solutions ("show work"). Appropriate partial credit will be given. Do not use a calculator to find limits or to sum infinite series.

13. Find the first four terms of the Maclaurin series (Taylor series around 0) of $f(x) = \dfrac{\cos x}{1-x}$.

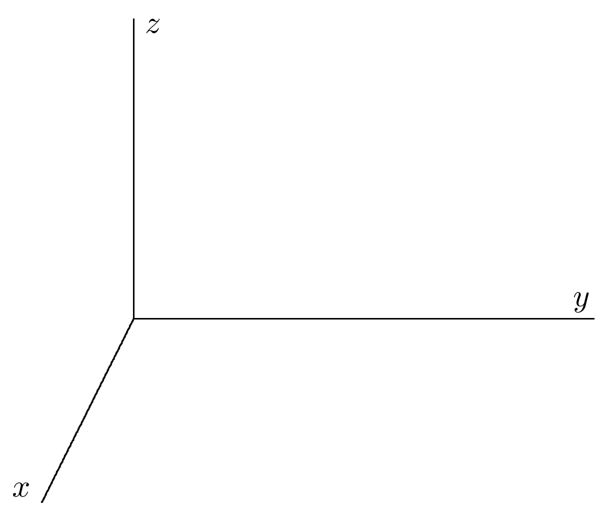
14. Starting from the Maclaurin series for $f(u) = e^u$, find the Maclaurin series for

$$g(x) = \int_0^x e^{-t^2}\,dt\,.$$

15. Let $\vec{a} = \langle 1, 2, 2 \rangle$ and $\vec{b} = \langle 4, 4, 0 \rangle$.

(a) Calculate $\vec{v} = \text{proj}_{\vec{a}}\vec{b}$, the vector projection of $\vec{b}$ onto $\vec{a}$.

(b) Sketch the three vectors $\vec{a}$, $\vec{b}$, and $\vec{v}$.



16. *(12 points)* Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{2^n n^2} \, .$$

(Full credit requires determining what happens at the endpoints of the interval.)

17. *Use the remainder estimate for the integral test* to show that $\sum_{n=1}^{\infty} \frac{1}{(n-1)^2+1}$ converges, and that the sum is a number between $1+\frac{\pi}{4}$ and $1+\frac{\pi}{2}$. Explain your reasoning clearly; a sketch will help.