DIRECTIONS:

1. The use of a cell phone, calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________________

Some integrals that may or may not be useful.

\[ \int \sec x \, dx = \ln | \sec x + \tan x | + C \]
\[ \int \csc x \, dx = \ln | \csc x - \cot x | + C \]
\[ \int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln | \sec x + \tan x | + C \]
\[ \int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln | \csc x - \cot x | + C \]
1. The sequence $a_n = \frac{(-1)^n n}{2n^2 + 5}$

(a) Converges to $-\frac{1}{2}$
(b) None of these.
(c) Converges to 0
(d) Diverges
(e) Converges to $\frac{1}{2}$

2. Which of the following is the form of the partial-fraction decomposition for the rational function $\frac{7}{(x + 3)^2(x^2 - 9)(x^2 + 16)}$?

(a) $\frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{Cx + D}{(x^2 - 9)} + \frac{Ex + F}{x^2 + 16}$

(b) $\frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{C}{(x + 3)^3} + \frac{D}{x - 3} + \frac{E}{x + 4} + \frac{F}{x - 4}$

(c) $\frac{A}{x + 3} + \frac{Bx + C}{(x + 3)^2} + \frac{Dx^2 + Ex + F}{(x + 3)^3} + \frac{G}{x - 3} + \frac{Hx + J}{x^2 + 16}$

(d) $\frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{C}{(x + 3)^3} + \frac{D}{x - 3} + \frac{Ex + F}{x^2 + 16}$

(e) None of these.

3. The sequence $a_n = 2\ln(3n + 5) - \ln(4n^2 + 1)$

(a) Diverges
(b) Converges to $\ln \left( \frac{9}{4} \right)$
(c) Converges to 0
(d) None of these
(e) Converges to $\ln \left( \frac{9}{4} \right)$
4. Compute \( \sum_{n=0}^{\infty} \frac{(-1)^n + 2^n}{6^n} = \)

(a) None of these.
(b) \( \frac{3}{10} \)
(c) \( \frac{33}{14} \)
(d) \( \frac{27}{10} \)
(e) \( \frac{5}{14} \)

5. The curve \( C \) is given by the equation \( x = 2 + 5y^2 \). Which integral gives the length of the curve from the point \((7, 1)\) to the point \((22, 2)\)?

(a) \( \int_{1}^{2} \sqrt{1 + 100y^2} \, dy \)
(b) \( \int_{7}^{22} \sqrt{1 + 10y} \, dy \)
(c) \( \int_{7}^{22} \sqrt{1 + 100y^2} \, dy \)
(d) \( \int_{1}^{2} \sqrt{1 + 10y} \, dy \)
(e) None of these.

6. Compute \( \int_{2}^{4} \frac{x^2}{x + 1} \, dx \)

(a) \( \frac{56}{3} - \ln(5) + \ln(3) \)
(b) \( 4 + \ln(5) - \ln(3) \)
(c) None of these
(d) \( 8 + \ln(5) - \ln(3) \)
(e) \( 4 - \ln(5) + \ln(3) \)
7. Which statement is true about the integral \( \int_{1}^{\infty} \frac{3 \sin^2 x}{x^2} \, dx \)?

(a) The integral diverges by comparison to \( \int_{1}^{\infty} \frac{3}{x^2} \, dx \)

(b) The integral diverges by comparison to \( \int_{1}^{\infty} \frac{1}{x} \, dx \)

(c) none of these.

(d) The integral converges by comparison to \( \int_{1}^{\infty} \frac{1}{x} \, dx \)

(e) The integral converges by comparison to \( \int_{1}^{\infty} \frac{3}{x^2} \, dx \)

8. Which of these series diverge by the Test for Divergence?

(a) \( \sum_{n=1}^{\infty} \frac{1}{5 - e^{-n}} \)

(b) None of these.

(c) \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \)

(d) \( \sum_{n=1}^{\infty} \sin \left( \frac{1}{n} \right) \)

(e) \( \sum_{n=1}^{\infty} \ln \left( \frac{n}{n + 7} \right) \)

9. After an appropriate substitution, the integral \( \int x^2 \sqrt{9 - x^2} \, dx \) is equivalent to which of the following?

(a) \( 9 \int \cos^2 \theta \, d\theta \)

(b) \( 27 \int \sin^2 \theta \cos \theta \, d\theta \)

(c) \( 81 \int \sec^3 \theta \tan^2 \theta \, d\theta \)

(d) \( 27 \int \sec^2 \theta \tan \theta \, d\theta \)

(e) \( 81 \int \sin^2 \theta \cos^2 \theta \, d\theta \)
10. The curve $C$ is given by the equations which are shown below goes from the point $(8, 4)$ to the point $(27, 9)$. Which integral gives the surface area obtained when rotating $C$ about the x-axis?

\[ x = t^3 \quad y = t^2 \]

(a) \( \int_{2}^{3} 2\pi t^3 \sqrt{9t^4 + 4t^2} \, dt \)

(b) \( \int_{8}^{27} 2\pi t^3 \sqrt{9t^4 + 4t^2} \, dt \)

(c) \( \int_{4}^{9} 2\pi t^2 \sqrt{9t^4 + 4t^2} \, dt \)

(d) \( \int_{2}^{3} 2\pi t^2 \sqrt{9t^4 + 4t^2} \, dt \)

(e) None of these.

11. Which of the following sequences is both bounded and decreasing?

(a) None of these.

(b) \( a_n = 1 - \frac{1}{n^2} \)

(c) \( a_n = \left(-\frac{1}{2}\right)^n \)

(d) \( a_n = 1 - \ln n \)

(e) \( a_n = \left(\frac{\pi}{4}\right)^n \)

12. Which of these substitutions would be used to evaluate \( \int \frac{1}{\sqrt{x^2 - 6x + 9}} \, dx \)?

(a) \( x - 3 = 2 \sec \theta \)

(b) \( x - 6 = \sqrt{5} \sec \theta \)

(c) none of these.

(d) \( x - 3 = 2 \tan \theta \)

(e) \( x^2 - 6x = \sqrt{5} \tan \theta \)
13. Let \( \sum_{n=1}^{\infty} a_n \) be a series whose \( n \)th partial sum is \( s_n = \frac{n}{2n+1} \). Find \( a_4 \).

(a) \( \frac{4}{9} \)
(b) None of these.
(c) \( \frac{1}{99} \)
(d) \( \frac{1}{63} \)
(e) \( \frac{1}{2} \)

14. After an appropriate substitution, the integral \( \int x^2 \sqrt{25 + 9x^2} \, dx \) is equivalent to which of the following?

(a) \( \frac{5^4}{3^3} \int \tan \theta \sec^4 \theta \, d\theta \)
(b) \( \frac{5^4}{3^3} \int \tan^2 \theta \sec^3 \theta \, d\theta \)
(c) \( \frac{5^3}{3^2} \int \sec \theta \tan^2 \theta \, d\theta \)
(d) None of these.
(e) \( \frac{5}{3^3} \int \tan^2 \theta \sec^3 \theta \, d\theta \)

15. The integral \( \int_{-2}^{0} \frac{2}{x^3} \, dx \)

(a) diverges to \( \infty \).
(b) converges to \( \frac{1}{4} \)
(c) converges to \( -\frac{1}{4} \)
(d) diverges to \( -\infty \).
(e) None of these.
PART II WORK OUT

**Directions**: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. Given the series \( \sum_{i=1}^{\infty} \left[ \cos \left( \frac{1}{i+2} \right) - \cos \left( \frac{1}{i} \right) \right] \).

(a) (5 points) Find \( s_n \), the nth partial sum. Simplify your answer.

(b) (3 points) Does the series converge or diverge? Justify your answer. Give the sum of the series if it converges.
17. (8 points) Compute \[ \int_{2}^{\infty} xe^{-2x} \, dx \]

18. (8 points) The curve \( x = y^3 \) is rotated about the \( y \)-axis for \( 1 \leq x \leq 27 \). Find the area of the resulting surface.
19. (8 points) Compute \( \int \frac{x^2}{(16 - x^2)^{3/2}} \, dx \).
20. (8 points) Compute \( \int \frac{x^4 + 3x^3 + 6x^2 + 7}{x^3(x^2 + 1)} dx \)