1. Compute \( \int_{1}^{\infty} \frac{1}{\sqrt{x} + x\sqrt{x}} \, dx \) by hand using \( w = \sqrt{x} \).
   (Check answer with calculator.)

2. Use the Comparison Theorem to determine whether the integral \( \int_{0}^{\pi} \frac{\sin^2 x}{\sqrt{x}} \, dx \) is convergent or divergent.
   Show your work by hand; check on your calculator.

3. Solve the initial value problem \( \frac{dy}{dx} = 2x\sqrt{1 - y^2} \), \( y(0) = 0 \), explicitly for \( y \) by hand. (Check with calc.)

4. Solve the differential equation \( x \ln x \, \frac{dy}{dx} + y = xe^x \) by hand. (Check with your calculator.)
Newton’s Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. A bottle of soda pop at room temperature (72°F) is placed in a refrigerator where the temperature is 44°F. After 30 minutes the soda pop has cooled to 61°F.

(a) Let $q$ be the temperature of the soda pop. State a differential equation and initial condition that models the description given. Then solve for $q$ in terms of $t$ (time). Use $\text{deSolve}$.

(b) When will the soda cool to 50°F?

An epidemic spreads at a rate jointly proportional to the number of infected people and the number of uninfected people. In an isolated town of 5000 inhabitants, 160 people have the disease at the beginning of the week and 1200 have it at the end of the week. How many days does it take for 80% of the population to become infected? Use $\text{deSolve}$.

Find the arc length of the curve $x = (y - 4)^{3/2}, \quad 5 \leq y \leq 8$. 

8. Find the arc length of the parametric curve
\[ x = 1 + 3t^2, \quad y = 4 + 2t^3, \quad 0 \leq t \leq 1. \]

9. Find the area of the surface obtained by rotating the curve
\[ x = 5 - y^2, \quad 0 \leq y \leq \sqrt{2}, \] about the x-axis.

10. Find the area of the surface obtained by rotating the parametric curve
\[ x = 2t^2 + \frac{1}{t}, \quad y = 8\sqrt{t}, \quad 1 \leq t \leq 3, \] about the x-axis.

11. Find the center of mass of the region bounded by
\[ y = x^3 - x \quad \text{and} \quad y = x^2 - 1. \] Density \( \delta = k \) is constant.

12. Masses \( p = [5, 4, 3, 6] \) are located at these points in the plane
\[ r = \begin{bmatrix} -4 & 2 \\ 0 & 5 \\ 3 & 2 \\ 1 & -2 \end{bmatrix}. \] (Here \( x \) is column 1 and \( y \) in column 2.) Find the center of mass of the system.
13. A rectangular tank 8 m long, 4 m wide, and 2 m high contains kerosene (density 820 kg/m³) to a depth of 1.5 m. Find the force on one vertical 4 × 2 meter end.

14. A triangular vertical plate is submerged in water (density 1000 kg/m³) as depicted below. Find the hydrostatic force against it.

15. Determine if the sequence \( a_n = \frac{n^2 \cos n}{1 + n^2} \) converges. (A sequence plot will help. Sketch it below.)

16. Find the limit of the positive recursive sequence \( a_1 = 1, \quad a_{n+1} = 1 + \frac{1}{a_n} \).
17. Determine if the series $\sum_{n=1}^{\infty} \frac{1}{1 + \left(\frac{3}{2}\right)^n}$ converges. Cite a test. If it converges, find its sum.

18. Express the following number as a ratio of positive integers (like I did in elementary school).

$$2.\overline{516} = 2.516516516\ldots$$

19. Show that the series converges by applying the Comparison Test or Integral Test (your choice). Compute its 10th partial sum, both exactly and approximately. Obtain an error estimate for the reminder via an improper integral.

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + n^3}$$

20. Determine whether the series $\sum_{n=1}^{\infty} \sin \left(\frac{1}{n}\right)$ converges or diverges. I recommend the Limit Comparison Test, which is especially tasty in this instance!
• Work by hand and/or calculator, showing your steps.
• Lengths are in centimeters; forces in newtons or lb.
• Write small and legibly. **Box final answers.**