DIRECTIONS:

1. The use of a cell phone, calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
PART I: Multiple Choice. 4 points each

1. Given the points $A(1,1,-1)$ and $B(2,0,1)$, which vector is orthogonal to the vector in the direction from $A$ to $B$?

   (a) $\langle 1,1,1 \rangle$
   (b) $\langle 2,2,0 \rangle$
   (c) $\langle 1,0,-2 \rangle$
   (d) None of these.
   (e) $\langle 1,-1,2 \rangle$

2. Which of these equations is a sphere that is centered at $(3,-2,2)$ and does not intersect the $xy$–plane?

   (a) $(x-3)^2 + (y+2)^2 + (z-2)^2 = 3$
   (b) $(x-3)^2 + (y+2)^2 + (z-2)^2 = 5$
   (c) none of these
   (d) $(x+3)^2 + (y-2)^2 + (z+2)^2 = 1$
   (e) $(x-3)^2 + (y-2)^2 + (z-2)^2 = 4$

3. Which of the following is the vector projection of the vector $\langle -2,3,1 \rangle$ onto the vector $\langle 1,1,2 \rangle$?

   (a) $\frac{3}{\sqrt{14}}$
   (b) $\left\langle \frac{1}{2}, \frac{1}{2}, 1 \right\rangle$
   (c) $\frac{3}{\sqrt{6}}$
   (d) $\left\langle -\frac{3}{7}, \frac{9}{14}, \frac{3}{14} \right\rangle$
   (e) None of these.
4. Suppose that $0 \leq a_n \leq b_n$ for every positive integer $n$. Which of the following statements is always true?

(a) If $\sum_{n=1}^{\infty} a_n$ is divergent, then so is $\sum_{n=1}^{\infty} b_n$.

(b) If $\lim_{n \to \infty} b_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(c) If $\sum_{n=1}^{\infty} b_n$ is divergent, then so is $\sum_{n=1}^{\infty} a_n$.

(d) None of these are always true.

(e) If $\sum_{n=1}^{\infty} a_n$ is convergent, then so is $\sum_{n=1}^{\infty} b_n$.

5. Suppose that the power series $\sum_{n=0}^{\infty} c_n (x - 2)^n$ has a radius of convergence of 5. Which of the following statements is true about the convergence/divergence of the following pair of series?

(I) $\sum_{n=0}^{\infty} (-1)^n c_n 4^n$

(II) $\sum_{n=0}^{\infty} c_n 7^n$

(a) (I) is divergent, (II) no conclusion may be made.

(b) (I) is convergent, (II) no conclusion may be made.

(c) (I) is divergent, (II) is convergent.

(d) Both series are convergent

(e) None of these.

6. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(4x + 1)^{2n}}{9^n}$.

(a) None of these.

(b) 0

(c) $\frac{9}{4}$

(d) $\infty$

(e) $\frac{3}{4}$
7. Which of the following is the third degree Taylor Polynomial for \( f(x) = x^4 + 5x^3 + 4x^2 + x + 7 \) centered at \( a = -1 \).

(a) \( T_3(x) = 6 + 4(x + 1) - 5(x + 1)^2 + 2(x + 1)^3 \)
(b) \( T_3(x) = 6 + 4(x + 1) - 5(x + 1)^2 + (x + 1)^3 \)
(c) \( T_3(x) = 7 + x + 4x^2 + 5x^3 \)
(d) \( T_3(x) = 6 + 4(x + 1) - 10(x + 1)^2 + 6(x + 1)^3 \)
(e) None of these.

8. Write \( f(x) = \frac{1}{9 - 4x^2} \) as a power series centered at 0.

(a) \( \sum_{n=0}^{\infty} (-1)^n \frac{9^n x^{2n}}{4^{n+1}} \)
(b) None of these.
(c) \( \sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{2n}}{9^{n+1}} \)
(d) \( \sum_{n=0}^{\infty} \frac{4^n x^{2n}}{9^{n+1}} \)
(e) \( \sum_{n=0}^{\infty} \frac{9^n x^{2n}}{4^{n+1}} \)

9. Which of the following statements is true for this pair of series.

(I) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}} \) (II) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{4/3}} \)

(a) None of these are true.
(b) Both series are absolutely convergent.
(c) (I) is convergent, but not absolutely convergent, (II) is absolutely convergent.
(d) Both series are convergent but not absolutely convergent.
(e) (I) is absolutely convergent, (II) is convergent, but not absolutely convergent.
10. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges to $s$. Based on the Alternating Series Estimation Theorem, which statement is true?

(a) $|R_7| = |s - s_7| \leq \frac{1}{7}$
(b) $|R_7| = |s - s_7| \leq \frac{1}{64}$
(c) None of these.
(d) $|R_7| = |s - s_7| \leq \frac{1}{8}$
(e) $|R_7| = |s - s_7| \leq \frac{1}{49}$

11. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{4^n(x-3)^n}{n!}$.

(a) $\frac{1}{4}$
(b) 0
(c) None of these.
(d) $\infty$
(e) 4

12. Which of the following is the 3rd degree Taylor Polynomial for $f(x) = x + e^{-2x}$ centered at $a = 0$?

(a) $T_3(x) = 1 - x + 4x^2 - 8x^3$
(b) None of these.
(c) $T_3(x) = 1 - x + 2x^2 - \frac{4}{3}x^3$
(d) $T_3(x) = 1 + 3x + 4x^2 - 8x^3$
(e) $T_3(x) = 1 + 3x + 2x^2 + \frac{4}{3}x^3$
13. Find the sum of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1}}{3^{2n}(2n+1)!} \)

(a) \( 5 \cos \left( \frac{5}{3} \right) \)
(b) None of these
(c) \( \frac{1}{3} \sin \left( \frac{5}{3} \right) \)
(d) \( \frac{1}{3} \arctan \left( \frac{5}{3} \right) \)
(e) \( 3 \sin \left( \frac{5}{3} \right) \)

14. Find the 15th derivative at \( x = 0 \) for \( f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n3^n} \)

(a) \( f^{(15)}(0) = \frac{1}{15(3^{15})} \)
(b) \( f^{(15)}(0) = \frac{14!}{3^{15}} \)
(c) \( f^{(15)}(0) = -\frac{1}{15(3^{15})} \)
(d) None of these.
(e) \( f^{(15)}(0) = -\frac{14!}{3^{15}} \)

15. Find the center and the radius of the sphere \((x-2)^2 + (y+3)^2 + z^2 + 8z + 7 = 0\)

(a) center: \((2, -3, -4)\), radius = 3
(b) center: \((-2, 3, 8)\), radius = 7
(c) None of these.
(d) center: \((2, -3, -4)\), radius = 9
(e) center: \((-2, 3, 4)\), radius = 3
PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (6 points) Determine if this series converges or diverges. Work must be shown to receive full credit.

\[ \sum_{n=9}^{\infty} \frac{n + 7}{n^{3/2} - n - 4} \]

17. (6 points) Evaluate the integral as an infinite series.

\[ \int \cos(5x^4) \, dx \]
18. (6 points) Find the Maclaurin series for \( f(x) \). Express your answer in summation notation.

\[
f(x) = \frac{x^4}{(1 + 5x)^2}
\]

19. (4 points) The function \( f(x) = \frac{1}{x^5} \) is approximated by \( T_3 \) centered at \( a = 5 \) on the interval \( 1 \leq x \leq 6 \). Estimate the maximum error, \( |R_3| \), for this approximation using Taylor’s Inequality.

\[
|R_n(x)| \leq \frac{M}{(n+1)!} |x - a|^{n+1} \quad \text{where} \quad |f^{(n+1)}(x)| \leq M \quad \text{For} \quad 1 \leq x \leq 6.
\]
20. (6 points) Find the Taylor series for \( f(x) = xe^x \) about \( a = 5 \). Express your answer in summation notation.

21. (6 points) Show that this series is converges absolutely.

\[
\sum_{n=2}^{\infty} \frac{(-1)^n}{n(ln n)^2}
\]
22. (6 points) Find the radius of convergence and the interval of convergence of the power series.

\[
\sum_{n=1}^{\infty} \frac{(5x - 4)^n}{n^2 4^n}
\]