MATH 152, FALL 2015
COMMON EXAM II - VERSION A

LAST NAME(print): __________________________ FIRST NAME(print): __________________________

INSTRUCTOR: ________________________________
SECTION NUMBER: __________________________

DIRECTIONS:
1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!
4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR
"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: __________________________________________

Some integrals that may or may not be useful.

\[ \int \sec x \; dx = \ln |\sec x + \tan x| + C \]
\[ \int \csc x \; dx = \ln |\csc x - \cot x| + C \]
\[ \int \sec^3 x \; dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C \]
\[ \int \csc^3 x \; dx = \frac{-1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C \]
PART I: Multiple Choice. 4 points each

1. Which of the following is the correct partial fraction decomposition for the function $f(x) = \frac{3x+7}{(x^2-9)^2(x^2+4)}$?

(a) $\frac{A}{(x-3)^2} + \frac{B}{(x+3)^2} + \frac{Cx+D}{x^2+4}$

(b) $\frac{Ax+B}{(x-3)^2} + \frac{Cx+D}{x^2+4}$

(c) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+3} + \frac{D}{(x+3)^2} + \frac{Ex+F}{x^2+4}$

(d) $\frac{Ax+B}{x^2-9} + \frac{Cx+D}{(x^2-9)^2} + \frac{Ex+F}{x^2+4}$

(e) $\frac{A}{(x-3)^2} + \frac{B}{(x+3)^2} + \frac{C}{x^2+4}$

2. Which of the following is the correct substitution to use when evaluating $\int \sqrt{x^2 - 8x + 20} \, dx$?

(a) $x - 4 = 36 \tan \theta$

(b) $x - 4 = 6 \tan \theta$

(c) $x - 4 = 4 \tan \theta$

(d) $x - 4 = 2 \tan \theta$

(e) $x^2 - 8x = \sqrt{20} \tan \theta$

3. Which of the following series diverges by the Test for Divergence?

(a) Only (I) diverges by the Test for Divergence.

(b) Only (II) diverges by the Test for Divergence.

(c) Both (I) and (II) diverge by the Test for Divergence.

(d) Both (I) and (II) converge.

(e) The Test for Divergence is inconclusive for both series.

$I) \sum_{n=1}^{\infty} \frac{1}{n} = e^1/e = 1 \rightarrow \text{Diverges}$

$II) \sum_{n=1}^{\infty} \frac{n^2}{n^4+3} = 0 \rightarrow \text{Test for Divergence Inconclusive}$

4. Which of the following integrals gives the surface area of the solid found by rotating the curve $x = y^3 + y$, $0 \leq y \leq 1$ about the x-axis? $\nu = y$

(a) $\int_{0}^{1} 2\pi y \sqrt{9y^4 + 4} \, dy$

(b) $\int_{0}^{1} 2\pi (y^3 + y) \sqrt{9y^4 + 6y^2 + 2} \, dy$

(c) $\int_{0}^{1} 2\pi (y^3 + y) \sqrt{9y^4 + 2} \, dy$

(d) $\int_{0}^{1} 2\pi y \sqrt{9y^4 + 6y^2 - 2} \, dy$

(e) $\int_{0}^{1} 2\pi y \sqrt{3y^2 + 2} \, dy$

\[
\int_{0}^{1} 2\pi y \sqrt{3y^2 + 1} \, dy
\]

\[
= \int_{0}^{1} 2\pi y \sqrt{9y^4 + 6y^2 + 2} \, dy
\]
5. Evaluate \( \int_2^t \frac{1}{(x-4)^3} \, dx \).

(a) \( \frac{1}{4} \)
(b) \( \frac{1}{8} \)
(c) \( -\frac{1}{8} \)
(d) \( -\frac{1}{4} \)
(e) The integral diverges.

\[
\begin{align*}
\int_2^t \frac{1}{(x-4)^3} \, dx &= -\frac{1}{2(t-4)^2} + \frac{1}{2(-2)^2} = -\frac{1}{2(t-4)^2} + \frac{1}{8} \\
\lim_{t \to 4^-} \left[ -\frac{1}{2(t-4)^2} + \frac{1}{8} \right] &= -\infty
\end{align*}
\]

6. Find the sum of the series \( \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) \).

(a) \( \frac{1}{2} \)
(b) \( \frac{3}{4} \)
(c) \( \frac{5}{6} \)
(d) 1
(e) 0

\[
S_n = \left( \frac{1}{1} - \frac{1}{3} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + \left( \frac{1}{3} - \frac{1}{5} \right) + \ldots + \left( \frac{1}{n-1} - \frac{1}{n+1} \right) + \left( \frac{1}{n} - \frac{1}{n+2} \right)
\]

\[
\lim_{n \to \infty} S_n = \frac{3}{2}
\]

7. Compute \( \int \frac{1}{\sqrt{1 + x^2}} \, dx \).

(a) \( \frac{\sqrt{1 + x^2}}{x} + C \)
(b) \( \ln \left| \sqrt{1 + x^2} + x \right| + C \)
(c) \( \ln \left| \frac{x}{\sqrt{1 + x^2}} + x \right| + C \)
(d) \( \frac{x}{\sqrt{1 + x^2}} + C \)
(e) \( \ln \left| \sqrt{1 + x^2} + x \right| + C \)

\[
\begin{align*}
\int \frac{1}{\sqrt{1 + x^2}} \, dx &= \int \frac{\sec^2 \theta}{\sqrt{1 + \tan^2 \theta}} \, d\theta \\
&= \int \sec^2 \theta \cdot \sec \theta \, d\theta \\
&= \int \sec \theta \, d\theta = \ln \left| \sec \theta + \tan \theta \right| + C
\end{align*}
\]

\[= \ln \left| \sqrt{x^2 + 1} + x \right| + C\]

8. Which of the following sequences is both bounded and increasing?

(a) \( a_n = \sin \left( \frac{n\pi}{2} \right) \) \( \rightarrow \) Bounded; Not monotonic
(b) \( a_n = \left( \frac{1}{4} \right)^n \) \( \rightarrow \) Bounded, decreasing
(c) \( a_n = 5^n \) \( \rightarrow \) Unbounded, increasing
(d) \( a_n = 2 - \frac{1}{n} \) \( \rightarrow \) \( \lim_{n \to \infty} a_n = 2 \); \( f(n) = \frac{2}{n^3} > 0 \) \( \rightarrow \) Increasing; Bounded
(e) \( a_n = 1 + \frac{1}{n} \) \( \rightarrow \) \( \lim_{n \to \infty} a_n = 1 \); \( f(n) = -\frac{1}{n^2} < 0 \) \( \rightarrow \) Decreasing; Bounded
9. Which of the following integrals gives the surface area of the solid found by rotating the curve $y = \sqrt{x+1}, 0 \leq x \leq 7$ about the $y$-axis? \[ y = \sqrt{x+1} \]

\[
\begin{align*}
\text{(a)} & \quad \int_1^7 2\pi (y^3 - 1) \sqrt{1 + 9y^4} \, dy \\
\text{(b)} & \quad \int_0^7 2\pi (y^3 - 1) \sqrt{1 + 9y^4} \, dy \\
\text{(c)} & \quad \int_1^7 2\pi y \sqrt{1 + 9y^4} \, dy \\
\text{(d)} & \quad \int_0^7 2\pi y \sqrt{1 + 9y^4} \, dy \\
\text{(e)} & \quad \int_1^7 y \sqrt{1 + 3y^2} \, dy
\end{align*}
\]

10. The recursive sequence defined by $a_1 = 3, a_{n+1} = 7 - \frac{10}{a_n}$ is increasing and bounded. Find the limit of the sequence.

\[ L = 7 - \frac{10}{L} \]

\[ L^2 = 7L - 10 \]

\[ (L-2)(L-5) = 0 \]

\[ L = 2, 5 \]

Since $a_n$ is increasing and $a_1 = 3$, then \[ L = 5 \]

11. Which of the following statements is true about the integral \[ \int_1^\infty \frac{4}{x^2 + \sqrt{x}} \, dx \]?

\begin{align*}
\text{(a)} & \quad \text{The integral diverges by comparison with } \int_1^\infty \frac{4}{x^2} \, dx \\
\text{(b)} & \quad \text{The integral diverges by comparison with } \int_1^\infty \frac{4}{\sqrt{x}} \, dx \\
\text{(c)} & \quad \text{The integral converges by comparison with } \int_1^\infty \frac{4}{x^2} \, dx \\
\text{(d)} & \quad \text{The integral converges by comparison with } \int_1^\infty \frac{4}{\sqrt{x}} \, dx \\
\text{(e)} & \quad \text{The convergence/divergence of the integral cannot be determined.}
\end{align*}

12. Find the sum of the series \[ \sum_{n=1}^{\infty} \frac{2^{n+1}}{(-5)^n} = \sum_{n=1}^{\infty} 2 \left( \frac{-2}{5} \right)^n \]

\[ 1 + r = -\frac{4}{5} \]

\[ r = -\frac{2}{5} \]

Since $|r| < 1$, geometric series converges

\[ \frac{-4/5}{1 - (-2/5)} = \frac{-4/1}{7/5} = -\frac{4}{7} \]
13. Evaluate \[ \int_2^4 \frac{x^2 + 1}{x - 1} \, dx \]

(a) 4
(b) 6 + 2 ln 3
(c) 6 + ln 3
(d) 2 + 2 ln 3
(e) 8 + 2 ln 3

\[ \int_2^4 \left( x + 1 + \frac{2}{x - 1} \right) \, dx = \int_2^4 \frac{1}{2} \, x^2 + x + 2 \ln |x - 1| \, dx \]

\[ = \left[ \frac{1}{3} x^3 + \frac{1}{2} x^2 + 2 \ln |x - 1| \right]_2^4 \]

\[ = \left( \frac{64}{3} + 8 + 2 \ln 3 \right) - \left( \frac{8}{3} + 2 + 2 \ln 1 \right) \]

\[ = 8 + 2 \ln 3 \]

14. Which of the following integrals results after performing an appropriate trigonometric substitution for the following integral?

\[ \int_0^{1/4} \frac{\sqrt{1 - 4x^2}}{x^2} \, dx \]

(a) \( \int_0^{\pi/6} \frac{4 \cos \theta}{\sin^2 \theta} \, d\theta \)
(b) \( \int_0^{\pi/2} \frac{4 \cot^2 \theta}{\sin^2 \theta} \, d\theta \)
(c) \( \int_0^{\pi/2} \frac{16 \cos \theta}{\sin^2 \theta} \, d\theta \)
(d) \( \int_0^{\pi/6} 2 \cot^2 \theta \, d\theta \)
(e) \( \int_0^{\pi/6} \frac{8 \cos \theta}{\sin^2 \theta} \, d\theta \)

\[ 2x = \sin \theta \]
\[ x = \frac{1}{2} \sin \theta \]
\[ dx = \frac{1}{2} \cos \theta \, d\theta \]

\[ \int_0^{\pi/6} \frac{\sqrt{1 - \sin^2 \theta}}{\frac{1}{4} \sin^2 \theta} \cdot \frac{1}{2} \cos \theta \, d\theta \]

\[ = \int_0^{\pi/6} \frac{2 \cos^2 \theta}{\sin^2 \theta} \, d\theta \]

15. Which of the following sequences converge?

(I) \( a_n = \cos \left( \frac{1}{n} \right) \)
(II) \( a_n = \frac{(1 - 3n)}{n + 1} \)
(III) \( a_n = \ln(n^2 + 1) - \ln(n) \)

(a) Only (I) converges
(b) Only (II) and (III) converge
(c) Only (I) and (III) converge
(d) Only (I) and (II) converge
(e) All three sequences diverge

I. \( \lim_{n \to \infty} \cos \left( \frac{1}{n} \right) = \cos 0 = 1 \)

II. \( \lim_{n \to \infty} \frac{(1 - 3n)}{n + 1} \) DNE; All sequence does not → 0.

III. \( \lim_{n \to \infty} \ln(n^2 + 1) - \ln(n) \)
\[ = \lim_{n \to \infty} \ln \left( \frac{n^2 + 1}{n} \right) \to \infty \]
PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Exx your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (7 points) Evaluate the following improper integral or show that it diverges.

\[ \int_{-\infty}^{0} xe^{x^2} \, dx \]

\[
\int_{t}^{0} xe^{x^2} \, dx \quad \text{substitute} \quad u=x^2, \quad du=2x \, dx,
\]

\[ \frac{1}{2} \int_{t}^{0} e^u \, du = \frac{1}{2} e^u \bigg|_{x=t}^{x=0} \]

\[ = \frac{1}{2} e^{x^2} \bigg|_{t}^{0} = \frac{1}{2} e^0 - \frac{1}{2} e^{t^2} = \frac{1}{2} - \frac{1}{2} e^{t^2} \]

\[ \lim_{t \to \infty} \left[ \frac{1}{2} - \frac{1}{2} e^{t^2} \right] = -\infty \]

Integral Diverges

17. (6 points) The series \( \sum_{n=1}^{\infty} c_n \) has partial sums given by \( s_n = \frac{n}{3n+1} \).

(a) Find \( a_3 \).

\[ a_3 = s_3 - s_2 = \frac{3}{10} - \frac{2}{7} = \frac{1}{70} \]

(b) Determine if the series converges or diverges. If it converges, give the sum.

\[ \lim_{n \to \infty} s_n = \frac{1}{3} \quad \text{Limit of partial sums is } \frac{1}{3}, \]

so sequence converges to a sum of \( \frac{1}{3} \).
18. (10 points) Compute \( \int \frac{4x^2 + 23}{(x-2)(x^2+9)} \, dx \)

\[
\frac{4x^2 + 23}{(x-2)(x^2+9)} = \frac{A}{x-2} + \frac{Bx + C}{x^2+9}
\]

\(4x^2 + 23 = A(x^2+9) + (Bx + C)(x-2)\)

If \(x=2\):
\[
16 + 23 = 13A + 0
\]

\[
39 = 13A \quad \Rightarrow \quad A = 3
\]

\[
4x^2 + 23 = 3(x^2+9) + (Bx + C)(x-2)
\]

\[
\frac{4x^2 + 23}{x-2} = \frac{3x^2 + 27}{x-2} + Bx^2 - 2Bx + Cx - 2C
\]

So:
\[
4 = 3 + B \quad \Rightarrow \quad B = 1
\]

\[
0 = -2B + C \quad \Rightarrow \quad C = 2B = 2
\]

\[
23 = 27 - 2C
\]

\[
\int \frac{3}{x-2} \, dx + \int \frac{x + 2}{x^2+9} \, dx
\]

\[
= \int \frac{3}{x-2} \, dx + \int \frac{x}{x^2+9} \, dx + \int \frac{2}{x^2+9} \, dx
\]

\[
= 3\ln|x-2| + \frac{1}{2} \ln|x^2+9| + \frac{2}{3} \arctan\left(\frac{x}{3}\right) + C
\]
19. (10 points) Compute the following integral. When possible, simplify any trig expressions completely.

\[ \int \frac{\sqrt{x^2 - 9}}{x^3} \, dx \]

\[ x = 3 \sec \theta \implies \sec \theta = \frac{x}{3} \]

\[ dx = 3 \sec \theta \tan \theta \, d\theta \]

\[ \int \frac{\sqrt{9\sec^2 \theta - 9}}{27\sec^3 \theta} \cdot 3\sec \theta \tan \theta \, d\theta \]

\[ = \int \frac{3 \tan \theta \cdot 3 \sec \theta \tan \theta}{27 \sec^3 \theta} \, d\theta = \frac{1}{3} \int \frac{\tan^2 \theta}{\sec^2 \theta} \, d\theta = \frac{1}{3} \int \sin^2 \theta \, d\theta \]

\[ = \frac{1}{3} \cdot \frac{1}{2} \int [1 - \cos 2\theta] \, d\theta \]

\[ = \frac{1}{6} \left[ \theta - \frac{1}{2} \sin 2\theta \right] + C \]

\[ = \frac{1}{6} \left[ \theta - \frac{1}{2} (2\sin \theta \cos \theta) \right] + C \]

\[ = \frac{1}{6} \left[ \arccsc \left( \frac{x}{3} \right) - \left( \frac{\sqrt{x^2 - 9}}{x} \right) \left( \frac{3}{x} \right) \right] + C \]
20. (7 points) Find the length of the curve \( x = \frac{4}{3}t^{3/2}, \ y = \frac{1}{2}t^2 - t, \ 1 \leq t \leq 2. \)

\[
\frac{dx}{dt} = \frac{12}{6} t^{1/2} = 2\sqrt{t} \quad \frac{dy}{dt} = t - 1
\]

\[
L = \int_1^2 \sqrt{(2\sqrt{t})^2 + (t-1)^2} \ dt
\]

\[
= \int_1^2 \sqrt{4t + t^2 - 2t + 1} \ dt
\]

\[
= \int_1^2 \sqrt{t^2 + 2t + 1} \ dt = \int_1^2 \sqrt{(t+1)^2} \ dt
\]

\[
= \int_1^2 (t+1) \ dt = \left[ \frac{1}{2}t^2 + t \right]_1^2
\]

\[
= (2 + 2) - (\frac{1}{2} + 1) = \sqrt{\frac{5}{2}}
\]