DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ______________________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 3 points each.

1. Find the equation of the sphere whose diameter has endpoints (2, 1, 6) and (4, -3, 8).
   (a) \((x + 3)^2 + (y - 1)^2 + (z + 7)^2 = 6\)
   (b) \((x - 3)^2 + (y + 1)^2 + (z - 7)^2 = 6\)
   (c) \((x - 3)^2 + (y + 1)^2 + (z - 7)^2 = 24\)
   (d) \((x - 3)^2 + (y - 1)^2 + (z - 7)^2 = 24\)
   (e) \((x - 6)^2 + (y + 2)^2 + (z - 14)^2 = 24\)

2. Which of the following statements is true of the series \(\sum_{n=1}^{\infty} \frac{n^3}{n^3 + n^2 + 1}\)?
   I. Diverges by comparison test with \(\sum_{n=1}^{\infty} \frac{1}{n}\)
   II. Diverges by limit comparison test with \(\sum_{n=1}^{\infty} \frac{1}{n}\)
   III. Diverges by Test for Divergence
   (a) Only I and II are true
   (b) Only I is true
   (c) Only II is true
   (d) Only III is true
   (e) None of I, II and III is true.

3. Suppose it is known that \(\sum_{n=0}^{\infty} c_n x^n\) converges when \(x = -3\) and diverges when \(x = 5\). Which of the following is certain to be true?
   (a) \(\sum_{n=0}^{\infty} c_n 3^n\) is convergent
   (b) \(\sum_{n=0}^{\infty} c_n (-5)^n\) is convergent
   (c) \(\sum_{n=0}^{\infty} (-1)^n c_n 4^n\) is divergent
   (d) \(\sum_{n=0}^{\infty} c_n 4^n\) is divergent
   (e) \(\sum_{n=0}^{\infty} (-1)^n c_n 7^n\) is divergent
4. \( \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\pi}{3}\right)^{2n+1}}{(2n+1)!} = \\
(a) \frac{1}{2} \\
(b) \frac{\sqrt{3}}{2} \\
(c) -\frac{\sqrt{3}}{2} \\
(d) \frac{1}{2} \\
(e) e^{2\pi/3} \)

5. Write \( f(x) = \frac{x^3}{8 + x} \) as a power series centered at 0.

(a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{8n+1} \)

(b) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+3}}{64n} \)

(c) \( \sum_{n=0}^{\infty} \left(\frac{x}{8}\right)^{n+3} \)

(d) \( \sum_{n=0}^{\infty} \frac{x^{n+3}}{8n+1} \)

(e) \( \sum_{n=0}^{\infty} \left(\frac{x}{8}\right)^{3n} \)

6. Find the third degree Taylor Polynomial, \( T_3(x) \), for \( f(x) = \cos(2x) \) at \( a = \frac{\pi}{6} \).

(a) \( T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) - 2 \left(x - \frac{\pi}{6}\right)^2 + 4\sqrt{3} \left(x - \frac{\pi}{6}\right)^3 \)

(b) \( T_3(x) = \frac{1}{2} - \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{6}\right) - \frac{1}{4} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{12} \left(x - \frac{\pi}{6}\right)^3 \)

(c) \( T_3(x) = \frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4} \left(x - \frac{\pi}{6}\right)^2 + \frac{\sqrt{3}}{3} \left(x - \frac{\pi}{6}\right)^3 \)

(d) \( T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) - \left(x - \frac{\pi}{6}\right)^2 + 2\sqrt{3} \left(x - \frac{\pi}{6}\right)^3 \)

(e) \( T_3(x) = \frac{1}{2} - \sqrt{3} \left(x - \frac{\pi}{6}\right) + \left(x - \frac{\pi}{6}\right)^2 + \frac{2\sqrt{3}}{3} \left(x - \frac{\pi}{6}\right)^3 \)
7. The region in \( \mathbb{R}^3 \) described by the equation \( y = 8 \) is:

(a) A line parallel to the \( x \)-axis.
(b) A plane parallel to the \( xz \) plane passing through the point \( (0, 8, 0) \).
(c) The point \( (0, 8, 0) \).
(d) A plane parallel to the \( yz \) plane passing through the point \( (0, 8, 0) \).
(e) A line parallel to the \( z \)-axis.

8. Using the Alternating Series Estimation Theorem, what is the smallest number of terms we must use to approximate

\[
\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}
\]

with error less than \( \frac{1}{120} \)?

(a) \( n = 11 \)
(b) \( n = 9 \)
(c) \( n = 10 \)
(d) \( n = 12 \)
(e) \( n = 13 \)

9. Find the radius of convergence, \( R \), and the interval of convergence, \( I \), for

\[
\sum_{n=1}^{\infty} \frac{(x + 5)^n n!}{3^n}
\]

(a) \( R = 3, I = (-8, -2) \)
(b) \( R = \infty, I = (-\infty, \infty) \)
(c) \( R = \infty, I = \{ -5 \} \)
(d) \( R = 3, I = (2, 8) \)
(e) \( R = 0, I = \{ -5 \} \)
10. For which of the following series is the Ratio Test inconclusive?

I) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 3} \)

II) \( \sum_{n=2}^{\infty} \frac{\ln n}{n} \)

III) \( \sum_{n=1}^{\infty} n4^n \)

(a) I and II only
(b) II only
(c) I only
(d) II and III only
(e) I, II, and III

11. Which of the following is a Maclaurin series for \( f(x) = x^3e^{-x^2} \)?

(a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!} \)

(b) \( \sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!} \)

(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!} \)

(d) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{n!} \)

(e) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{n!} \)

12. \( \sum_{n=1}^{\infty} \frac{\arctan(n) + 4}{n^8} \) is:

(a) Convergent by the Comparison Test with \( \sum_{n=1}^{\infty} \frac{1}{n^8} \)

(b) Convergent by the Comparison Test with \( \sum_{n=1}^{\infty} \frac{\pi}{2n^8} \)

(c) Divergent by the Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{1}{n^8} \)

(d) Divergent by the Test for Divergence

(e) Divergent by the Integral Test
13. \[ \int \cos(2x^2) \, dx = \]

(a) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+3}}{(4n + 3)(2n + 1)!} \]

(b) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+4}}{(4n + 4)(2n)!} \]

(c) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{4n+2}}{(2n + 1)(2n)!} \]

(d) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+2}}{(2n + 1)(2n)!} \]

(e) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{4n+1}}{(4n + 1)(2n)!} \]

14. \[ \sum_{n=0}^{\infty} 5(-6)^n x^{2n} = \]

(a) \[ \frac{5}{1 + 6x^2}, \text{ where } |x| < \frac{1}{\sqrt{6}} \]

(b) \[ -\frac{5}{1 + 6x^2}, \text{ where } |x| < \sqrt{6} \]

(c) \[ \frac{5}{(1 - 6x)^2}, \text{ where } |x| < \sqrt{6} \]

(d) \[ -\frac{5}{1 + 6x^2}, \text{ where } |x| < \frac{1}{\sqrt{6}} \]

(e) \[ \frac{5}{(1 + 6x)^2}, \text{ where } |x| < \sqrt{6} \]

15. Find \( f^{(23)}(3) \), that is, the 23\textsuperscript{rd} derivative of \( f(x) \) at \( x = 3 \), if \( f(x) = \sum_{n=0}^{\infty} \frac{(-3)^n}{(2n)!} (x - 3)^n \).

(a) \[ \frac{(-3)^{24}(23)!}{(46)!} \]

(b) \[ \frac{(-3)^{25}(24)!}{(46)!} \]

(c) \[ \frac{(-3)^{25}}{(46)!} \]

(d) \[ \frac{(-3)^{25}(23)!}{(46)!} \]

(e) 0
16. For the power series \[ \sum_{n=2}^{\infty} \frac{(x + 1)^n}{(-5)^n \ln(n)} \]:

a.) (4 pts) Find the radius of convergence.

b.) (8 pts) Find the interval of convergence. You must test the endpoints for convergence.
17. For \( f(x) = \frac{2}{x^2} \):

a.) (9 pts) Find the Taylor series at \( a = 3 \).

b.) (3 pts) Find the radius of convergence for the Taylor series found in part a.).
18. (8 pts) Consider \( \sum_{n=2}^{\infty} \frac{(-1)^n n^3}{n^4 - 1} \). Determine whether the series converges absolutely, converges but not absolutely, or diverges. Fully support your conclusion.

19. Consider \( \sum_{n=1}^{\infty} \frac{1}{(n + 2)(\ln(n + 2))^2} \).

a.) (5 pts) Prove the series converges.

b.) (4 pts) If we used \( s_3 \), the third partial sum, to approximate the sum \( s \) of the series, use the Remainder Estimate for the Integral Test to find an upper bound on the remainder, \( R_3 = s - s_3 \).
20. (i) (9 pts) Find a power series about zero for \( f(x) = \ln(9 + x^2) \).

(ii) (3 pts) What is the radius of convergence of the series above?

(iii) (2 pts) Using (i), find a power series about zero for \( g(x) = x^3 \ln(9 + x^2) \).