Exam1 version B

Wednesday, November 3, 2021 10:29 AM

2021cM152
X18sols

MATH 152, FALL 2021
COMMON EXAM I - VERSION B KEY

LAST NAME(print): ____________ FIRST NAME(print): ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2 (Problems 16-19), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*
6. **Again. The use of a calculator, laptop or computer is prohibited.**

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: ____________

FOR INSTRUCTOR USE ONLY

Question	Points Awarded	Points
1-15	ScanTron	60
16		9
17		11
18		8
19		12
TOTAL		100

Part 1: Multiple Choice (4 points each)

1. Evaluate $\int x^3\sqrt{x^2+1}\,dx$.
   (a) $\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$ key
   (b) $3x^2\sqrt{x^2+1} + \frac{x^4}{\sqrt{x^2+1}} + C$
   (c) $\frac{2}{5}(x^2+1)^{5/2} - \frac{2}{3}(x^2+1)^{3/2} + C$
   (d) $\frac{1}{3}(x^2+1)^{3/2} + C$
   (e) None of these

$\int x^2 \cdot \sqrt{x^2+1}\,x\,dx$. Let $u = x^2+1$, $du = 2x\,dx$, $\frac12 du = x\,dx$ and $x^2 = u-1$
$= \int (u-1)\sqrt{u}\cdot\frac12 du = \frac12\int (u^{3/2}-u^{1/2})\,du = \frac12\left[\frac25 u^{5/2} - \frac23 u^{3/2}\right]+C = \frac15(x^2+1)^{5/2} - \frac13(x^2+1)^{3/2}+C$

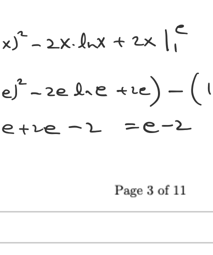
2. Evaluate $\int \frac{1+x}{1+x^2}\,dx$.
   (a) $\frac12\ln(1+x^2)+C$
   (b) $\frac12\ln(1+x^2) + \arctan x + C$ key
   (c) $\frac12\ln(1+x^2) + C$
   (d) $\ln(1+x^2)+C$
   (e) $\arctan x + \arcsin(x^2) + C$

$= \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}dx$. Let $u = 1+x^2$, $du = 2x\,dx$, $\frac12 du = x\,dx$
$= \arctan x + \frac12 \ln(1+x^2) + C$

3. Compute $\int_1^e (\ln x)^2\,dx$.
   (a) $e-2$ key
   (b) $\frac12 - 1$
   (c) $\frac{e}{2} - 2$
   (d) $e-1$
   (e) 1

by parts: $u = (\ln x)^2$, $du = 2\ln x\cdot\frac1x dx$, $dv = 1\,dx$, $v = x$
$= x(\ln x)^2\Big|_1^e - \int_1^e 2\ln x\,dx$; $u = 2\ln x$, $du = \frac2x dx$, $dv = dx$, $v = x$
$= x(\ln x)^2 - 2x\ln x + 2x\Big|_1^e$
$= (e(\ln e)^2 - 2e\ln e + 2e) - (1(\ln 1)^2 - 2(1)\ln(1) + 2(1))$
$= e - 2e + 2e - 2 = e - 2$

4. Which of the following gives the volume of the solid found by rotating the region bounded by the curves $y = 7 - x^2$ and $y = 3$ about the line $y = 0$?
   (a) $\int_3^7 2\pi y\sqrt{4-y}\,dy$
   (b) $\int_{-2}^{2} \pi(4-x^2)^2\,dx$
   (c) $\int_{-2}^{2} \pi[(7-x^2)^2 - 9]\,dx$ key
   (d) $\int_{-2}^{2} 2\pi x(7-x^2)\,dx$
   (e) $\int_{-2}^{2} \pi[(7-x^2) - 3]\,dx$



$V = \int_{-2}^{2}\pi[R^2 - r^2]\,dx = \int_{-2}^{2}\pi[(7-x^2)^2 - 3^2]\,dx$

Intersection: $7 - x^2 = 3$, $4 = x^2$, $\therefore x = \pm 2$

5. A spring has a natural length of 2 m. If a force of 12 N is required to hold the spring to a length of 4 m, find the work done to stretch the spring from 3 m to 5 m.
   (a) 30 J
   (b) 27 J
   (c) 12 J
   (d) 24 J key
   (e) 6 J

$F = kx \Rightarrow 12 = k(4-2)$, $\therefore k = 6$, $\therefore F = 6x$
Work done: $W = \int_1^3 f(x)\,dx = \int_1^3 6x\,dx = 3x^2\Big|_1^3 = 3[9-1] = 24$ J

6. Evaluate $\int_{\sqrt2}^{\sqrt5} \frac{4x}{x^2-1}\,dx$
   (a) $\ln 3 - 1$
   (b) $2\ln 2 - 2\ln\sqrt2$
   (c) $2\ln 3 - 2$
   (d) $2\ln 2$
   (e) $2\ln 3$ key

Let $u = x^2 - 1 \Rightarrow du = 2x\,dx \Rightarrow \frac12 du = x\,dx$
and $x = \sqrt2 \Rightarrow u = (\sqrt2)^2 - 1 = 1$; $x = \sqrt5 \Rightarrow u = (\sqrt5)^2 - 1 = 4$
$= \int_1^4 \frac4u\cdot\frac12 du = 2\int_1^4 \frac1u du = 2[\ln|u|]_1^4 = 2[(\ln(4)) - (\ln(1))] = 4\ln 2$

7. If f is continuous and $\int_0^{16} f(x)\,dx = 8$, find $\int_0^4 xf(x^2)\,dx$.
   (a) 2
   (b) 4 key
   (c) 8
   (d) 16
   (e) 64

Let $u = x^2 \Rightarrow du = 2x\,dx \Rightarrow \frac12 du = x\,dx$
And $x = 0 \Rightarrow u = 0$; $x = 4 \Rightarrow u = 4^2 = 16$
$= \int_0^{16} f(u)\cdot\frac12 du = \frac12\int_0^{16} f(u)\,du = \frac12\cdot 8 = 4$

8. Evaluate $\int \tan^3(x)\sec^3(x)\,dx$.
   (a) $\frac14\tan^4 x - \frac12\sec^2 x + C$
   (b) $\frac14\sec^4 x - \frac{1}{10}\tan^{10} x + C$
   (c) $\frac12\sec^2 x - \frac13\tan^3 x + C$
   (d) $\frac15\sec^5 x - \frac13\tan^3 x + C$
   (e) $\frac15\sec^5 x - \frac13\sec^3 x + C$ key

$= \int \tan^2 x\sec^2 x\cdot\tan x\sec x\,dx = \int(\sec^2 x - 1)\sec^2 x\cdot\tan x\sec x\,dx$
Let $u = \sec x \Rightarrow du = \sec x\tan x\,dx$
$= \int (u^2-1)\cdot u^2\,du = \int (u^4 - u^2)\,du = \frac15 u^5 - \frac13 u^3 + C = \frac15\sec^5 x - \frac13\sec^3 x + C$

9. A 20 ft rope weighing 0.1 lb/ft is hanging down the side of a 20 ft building. There is a 5 lb bucket attached to the rope. How much work is required to pull the rope with the bucket 2 ft up the side of the building?
   (a) 8.8 ft − lb
   (b) 14.2 ft − lb
   (c) 14 ft − lb
   (d) 13.8 ft − lb key
   (e) 120 ft − lb

Total weight $= 5 + 0.1\times(20 - x) = 5 + 2 - 0.1x = 7 - 0.1x$
Work done: $W = \int_0^2 (7 - 0.1x)\,dx = 7x - \frac{0.1}{2}x^2\Big|_0^2 = (14 - 0.2) - (0) = 13.8$ ft-lb

10. Compute $\int_0^2 (x^2+3)e^{-x}\,dx$.
   (a) $-\frac{8}{e^2} + 5$ key
   (b) $-\frac{8}{e}$
   (c) $\frac{8}{e} + 1$
   (d) $\frac{8}{e} - 1$
   (e) $-\frac{2}{e} + 5$

by parts: $u = x^2+3$, $dv = e^{-x}dx$, $du = 2x\,dx$, $v = -e^{-x}$
$= -(x^2+3)e^{-x}\Big|_0^2 - \int_0^2 -2xe^{-x}\,dx$; $u = 2x$, $dv = e^{-x}dx$, $du = 2\,dx$, $v = -e^{-x}$
$= 2xe^{-x}\Big|_0^2 - \int_0^2 2e^{-x}dx = 2x\cdot e^{-x}\Big|_0^2 + 2e^{-x}\Big|_0^2$
$= -(x^2+3)e^{-x} - 2x\cdot e^{-x} - 2\cdot e^{-x}\Big|_0^2$
$= (-4e^{-2} - 2\cdot e^{-2}\cdot 2 \cdot e^{-2}) - (-3 - 2)$
$= -\frac{8}{e^2} + 5$

[illegible]

$= \int \sin^7\theta\cdot(1-\sin^2\theta)^2\cdot\cos\theta\,d\theta$

Let $u = \sin\theta \Rightarrow du = \cos\theta\,d\theta$

$= \int u^7(1-u^2)^2\cdot du$
$= \int u^7(1-2u^2+u^4)\,du$
$= \int (u^7 - 2u^9 + u^{11})\,du$
$= \frac18 u^8 - \frac15 u^{10} + \frac1{12}u^{12} + C$
$= \frac18\sin^8\theta - \frac15\sin^{10}\theta + \frac1{12}\sin^{12}\theta + C$

19. (12 pts) A spherical tank with radius 5 m is half full of a liquid that has a density of 1000 kg/m^3. The tank has a 1 m spout at the top. Set up an integral to find the work required to pump all the water out of the spout. (Use 9.8 m/s^2 for g.)

Note 1. Do NOT evaluate your integral.

Note 2. Clearly indicate in the picture below where you are placing your axis and which direction is positive.

key:

(a) If the center of tank is 0, and going up is positive direction: $W = 9800\pi\int_{-5}^{0}(25-y^2)(6-y)\,dy$

(b) If the bottom of tank is 0, and going up is positive direction: $W = 9800\pi\int_{0}^{5}(25-(5-y)^2)(11-y)\,dy$

(c) If the top of tank is 0, and going down is positive direction: $W = 9800\pi\int_{5}^{10}(25-(y-5)^2)(y+1)\,dy$

(d) If the top of tank is 0, and going down is negative direction: $W = 9800\pi\int_{-10}^{-5}(25-(y+5)^2)(1-y)\,dy$