



MATH 152, FALL 2021
 COMMON EXAM I - VERSION B KEY

LAST NAME(print): _____ FIRST NAME(print): _____

INSTRUCTOR: _____

SECTION NUMBER: _____

DIRECTIONS:

- The use of a calculator, laptop or computer is prohibited.
- TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
- In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!
- In Part 2 (Problems 16-19), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- Be sure to write your name, section number and version letter of the exam on the ScanTron form.
- Again. The use of a calculator, laptop or computer is prohibited.

THE AGGIE DOES NOT LIE, CHEAT, OR STEAL

"An Aggie does not lie, cheat, or steal, or tolerate those who do."

Signature: _____

FOR INSTRUCTOR USE ONLY

Question	Points Awarded	Points
1-15	ScanTron	60
16		9
17		11
18		8
19		12
TOTAL		100

Part 1: Multiple Choice (4 points each)

1. Evaluate $\int x^2 \sqrt{x^2+1} dx$.
- (a) $\frac{1}{3}(x^2+1)^{3/2} - \frac{1}{5}(x^2+1)^{5/2} + C$ key
 (b) $3x^2 \sqrt{x^2+1} + \frac{2}{5}(x^2+1)^{5/2} + C$
 (c) $\frac{2}{3}(x^2+1)^{3/2} - \frac{2}{5}(x^2+1)^{5/2} + C$
 (d) $\frac{1}{3}(x^2+1)^{3/2} + C$
 (e) None of these
- Solution:* $\int x^2 \cdot \sqrt{x^2+1} dx = \int x \cdot x \cdot \sqrt{x^2+1} dx$
 Let $u = x^2+1$
 $du = 2x dx$
 $\frac{1}{2} du = x dx$
 and $x^2 = u-1$
 $= \int (u-1) \cdot \sqrt{u} \cdot \frac{1}{2} du = \frac{1}{2} \int (u^{3/2} - u^{1/2}) du = \frac{1}{2} [\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2}] + C = \frac{1}{5} (x^2+1)^{5/2} - \frac{1}{3} (x^2+1)^{3/2} + C$

2. Evaluate $\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$
- (a) $\frac{1}{2} \ln|1+x^2| + C$
 (b) $\frac{1}{2} \ln|1+x^2| + \arctan x + C$ key
 (c) $\frac{1}{2} \ln|1+x^2| + C$
 (d) $\ln|1+x^2| + C$
 (e) $\arctan x + \arcsin(x^2) + C$
- Solution:* $\int \frac{x}{1+x^2} dx = \frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \ln|1+x^2| + C$
 $\int \frac{1}{1+x^2} dx = \arctan x + C$
 Total: $\arctan x + \frac{1}{2} \ln|1+x^2| + C$

3. Compute $\int_1^e (\ln x)^2 dx$.
- (a) $e-2$ key
 (b) $\frac{1}{2} - 1$
 (c) $\frac{2}{e} - 2$
 (d) $e-1$
 (e) 1
- Solution:* $\int_1^e (\ln x)^2 dx = x(\ln x)^2 - \int 2x \ln x dx = x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2(x \ln x - \int 1 dx) = x(\ln x)^2 - 2x \ln x + 2x$
 Evaluate from 1 to e: $(e \ln^2 e - 2e \ln e + 2e) - (1 \ln^2 1 - 2 \ln 1 + 2) = e^2 - 2e + 2 - 2 = e^2 - 2e = e(e-2)$

4. Which of the following gives the volume of the solid found by rotating the region bounded by the curves $y = 7-x^2$ and $y = 3$ about the line $y = 0$?
- (a) $\int_2^7 2\pi y \sqrt{4-y} dy$
 (b) $\int_{-2}^2 \pi(4-x^2)^2 dx$
 (c) $\int_{-2}^2 \pi((7-x^2)^2 - 9) dx$ key
 (d) $\int_2^7 2\pi x(7-x^2) dx$
 (e) $\int_{-2}^2 \pi((7-x^2)-3) dx$
- Solution:* Volume = $\int_{-2}^2 \pi [R^2 - r^2] dx = \int_{-2}^2 \pi [(7-x^2)^2 - 3^2] dx$

5. A spring has a natural length of 2 m. If a force of 12 N is required to hold the spring to a length of 4 m, find the work done to stretch the spring from 3 m to 5 m.
- (a) 30 J
 (b) 27 J
 (c) 12 J
 (d) 24 J key
 (e) 6 J
- Solution:* $F = kx \Rightarrow 12 = k(4-2) \Rightarrow k = 6$
 Work = $\int_3^5 6x dx = 3x^2 \Big|_3^5 = 3(25-9) = 24 J$

6. Evaluate $\int_1^2 \frac{dx}{x^2-1}$
- (a) $\ln 3 - 1$
 (b) $2 \ln 2 - 2 \ln \sqrt{2}$
 (c) $2 \ln 3 - 2$
 (d) $2 \ln 2$
 (e) $2 \ln 3$ key
- Solution:* Let $u = x^2-1$
 $du = 2x dx$
 $\frac{1}{2} du = x dx$
 $\int \frac{1}{u} \cdot \frac{1}{2} du = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2-1| + C$
 Evaluate from 1 to 2: $\frac{1}{2} [\ln(3) - \ln(0)]$ (Note: limit as x approaches 1 from right)

7. If f is continuous and $\int_0^6 f(x) dx = 8$, find $\int_0^4 f(x^2) dx = \int_0^4 f(u) \cdot \frac{1}{2} du$
- (a) 2
 (b) 4 key
 (c) 8
 (d) 16
 (e) 64
- Solution:* Let $u = x^2$
 $du = 2x dx \Rightarrow \frac{1}{2} du = x dx$
 When $x=0, u=0$
 When $x=2, u=4$
 $\int_0^4 f(x^2) dx = \int_0^4 f(u) \cdot \frac{1}{2} du = \frac{1}{2} \int_0^4 f(u) du = \frac{1}{2} \cdot 8 = 4$

8. Evaluate $\int \tan^3(x) \sec^3(x) dx = \int \tan^2(x) \sec^2(x) \cdot \tan(x) \cdot \sec(x) dx$
- (a) $\frac{1}{3} \tan^3 x - \frac{1}{5} \sec^5 x + C$
 (b) $\frac{1}{4} \sec^4 x - \frac{1}{2} \tan^3 x + C$
 (c) $\frac{1}{4} \sec^4 x - \frac{1}{2} \tan^3 x + C$
 (d) $\frac{1}{4} \sec^4 x - \frac{1}{2} \tan^3 x + C$ key
 (e) $\frac{1}{4} \sec^4 x - \frac{1}{2} \sec^3 x + C$
- Solution:* Let $u = \sec x$
 $du = \sec x \tan x dx$
 $\int \tan^2(x) \sec^2(x) \cdot \tan(x) \cdot \sec(x) dx = \int (u^2-1) \cdot u^2 \cdot du = \int (u^4 - u^2) du = \frac{1}{5} u^5 - \frac{1}{3} u^3 + C = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$

9. A 20 ft rope weighing 0.1 lb/ft is hanging down the side of a 20 ft building. There is a 5 lb bucket attached to the rope. How much work is required to pull the rope with the bucket 2 ft up the side of the building?
- (a) 6.8 ft-lb
 (b) 14.2 ft-lb
 (c) 14 ft-lb
 (d) 13.8 ft-lb key
 (e) 120 ft-lb
- Solution:* Total weight = $5 + 0.1(20-x) = 5 + 2 - 0.1x = 7 - 0.1x$
 Work = $\int_0^2 (7 - 0.1x) dx = 7x - 0.05x^2 \Big|_0^2 = 14 - 0.2 = 13.8 ft-lb$

10. Compute $\int_0^1 (x^2+3)e^{-x} dx$.
- (a) $-\frac{8}{e} + 5$ key
 (b) $-\frac{8}{e}$
 (c) $\frac{2}{e} + 1$
 (d) $-\frac{2}{e} + 1$
 (e) $-\frac{2}{e} + 5$
- Solution:* $\int_0^1 (x^2+3)e^{-x} dx = -\left[(x^2+3)e^{-x} \right]_0^1 + \int_0^1 2xe^{-x} dx = -\left[(1+3)e^{-1} - 3 \right] + \int_0^1 2xe^{-x} dx$
 $\int_0^1 2xe^{-x} dx = -2xe^{-x} + 2e^{-x} \Big|_0^1 = -2e^{-1} + 2e^{-1} - 2 = -2$
 Total: $-4e^{-1} + 3 - 2 = -\frac{4}{e} + 1$

11. Calculate the area of the region bounded by the curves $4x + y^2 = 12$ and $y = x$.
- (a) 21
 (b) 22 key
 (c) $\frac{8\pi}{3}$
 (d) $\frac{2\pi}{3}$
 (e) $\frac{2\pi}{3}$
- Solution:* $4x + y^2 = 12 \Rightarrow x = 3 - \frac{1}{4}y^2$
 $y = x \Rightarrow y = 3 - \frac{1}{4}y^2 \Rightarrow 4y = 12 - y^2 \Rightarrow y^2 + 4y - 12 = 0 \Rightarrow (y+6)(y-2) = 0 \Rightarrow y = 2$
 Area = $\int_{-2}^2 [3 - \frac{1}{4}y^2 - y] dy = \left[3y - \frac{1}{12}y^3 - \frac{1}{2}y^2 \right]_{-2}^2 = \left[6 - \frac{2}{3} - 2 \right] - \left[-6 + \frac{2}{3} - 2 \right] = \frac{8\pi}{3}$

13. Find the volume of the solid found by rotating the region bounded by the curves $y = -x^2 + 2x$ and $y = 0$ about the y -axis.
- (a) $\frac{9\pi}{8}$
 (b) $\frac{3\pi}{8}$
 (c) $\frac{3\pi}{4}$ key
 (d) $\frac{3\pi}{2}$
 (e) $\frac{3\pi}{4}$
- Solution:* Volume = $\int_0^2 2\pi \cdot x \cdot (-x^2+2x) dx = 2\pi \int_0^2 (-x^3 + 2x^2) dx = 2\pi \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 \right]_0^2 = 2\pi \left[-4 + \frac{16}{3} \right] = 2\pi \left(\frac{4}{3} \right) = \frac{8\pi}{3}$

14. Evaluate $\int \sin^3(x) dx = \int_0^{\pi/2} 2 \cdot \sin^2(x) \cdot \cos(x) \cdot (-\sin(x)) dx = -2 \int_0^{\pi/2} \sin^2(x) \cos(x) dx$
- (a) $\frac{1}{3} \cos^3 x + C$
 (b) $\frac{1}{3} x - \frac{1}{2} \sin(2x) + C$ key
 (c) $\frac{1}{3} x - \frac{1}{2} \sin(2x) + C$
 (d) $\frac{1}{3} x + \frac{1}{2} \sin x + C$
 (e) $\frac{1}{3} x + \frac{1}{2} \sin x + C$
- Solution:* Let $u = \sin x$
 $du = \cos x dx$
 $\int \sin^2(x) \cos(x) dx = \int (u^2-1) du = \frac{1}{3} u^3 - u + C = \frac{1}{3} \sin^3 x - \sin x + C$

15. Which of the following represents the area bounded by the curves $y = x^2 - 4$ and $y = -x^2 - 2x$ on the interval $-3 \leq x \leq 1$.
- (a) $\int_{-3}^1 (-2x^2 - 2x + 4) dx$
 (b) $\int_{-3}^1 (-2x^2 - 2x + 4) dx$
 (c) $\int_{-3}^1 (2x^2 + 2x - 4) dx$
 (d) $\int_{-3}^1 (-2x^2 - 2x + 4) dx + \int_{-3}^1 (2x^2 + 2x - 4) dx$
 (e) $\int_{-3}^1 (2x^2 - 2x - 4) dx + \int_{-3}^1 (-2x^2 - 2x + 4) dx$ key
- Solution:* Area = $\int_{-3}^1 [(x^2-4) - (-x^2-2x)] dx = \int_{-3}^1 (2x^2 + 2x - 4) dx$

Part 2: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and for your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (9 pts) Consider the region R bounded by $y = x^2$, $y = -x + 2$, $x = 0$, and $x = 1$.
- (a) (3 pts) Sketch the region R .
- Solution:*

- (b) (3 pts) Set up the integral that gives the volume obtained by revolving the region R about the x -axis using the method of cylindrical shells. DO NOT EVALUATE THE INTEGRAL.
- key: $\int_0^1 2\pi [(1-x)^2 - (x^2)^2] dx$
- Solution:* Volume = $\int_0^1 2\pi [R^2 - r^2] dx = \int_0^1 2\pi [(1-x)^2 - x^4] dx$

- (c) (3 pts) Set up the integral that gives the volume obtained by revolving the region R about the line $x = 1$ using the method of cylindrical shells. DO NOT EVALUATE THE INTEGRAL.
- key: $\int_0^1 2\pi(1-x)(-x+2-x^2) dx$
- Solution:* Volume = $\int_0^1 2\pi r \cdot h dx = \int_0^1 2\pi(1-x)[(-x+2)-x^2] dx$

17. (11 pts) The base of a solid is the region bounded by the curve $y = 5 - x^2$ and the x -axis. Cross-Sections perpendicular to the y -axis are rectangles with height equal to twice the base. Find the volume of this solid.
- key: 100
- Solution:* $y = 5 - x^2 \Rightarrow x^2 = 5 - y \Rightarrow x = \sqrt{5-y}$
 Base = $2x = 2\sqrt{5-y}$
 Area = $b \cdot h = b \cdot 2b = 2b^2 = 2(2\sqrt{5-y})^2 = 8(5-y)$
 Volume = $\int_0^5 8(5-y) dy = 8 \left[5y - \frac{1}{2}y^2 \right]_0^5 = 8 \left[25 - \frac{25}{2} \right] = 8 \cdot \frac{25}{2} = 100$

18. (8 pts) Compute $\int \sin^3 \theta \cos^4 \theta d\theta$.
- key: $\frac{1}{8} \sin^8 \theta - \frac{1}{5} \sin^{10} \theta + \frac{1}{12} \sin^{12} \theta + C$
- Solution:* $\int \sin^2 \theta \cdot \sin \theta \cdot \cos^4 \theta d\theta = \int (1 - \sin^2 \theta) \sin \theta \cos^4 \theta d\theta = \int \cos^4 \theta d\theta - \int \sin^2 \theta \cos^4 \theta d\theta$
 $\int \cos^4 \theta d\theta = \int \cos^2 \theta \cdot \cos^2 \theta d\theta = \int (1 - \sin^2 \theta) \cos^2 \theta d\theta = \int \cos^2 \theta d\theta - \int \sin^2 \theta \cos^2 \theta d\theta$
 $\int \cos^2 \theta d\theta = \frac{1}{2} \int (1 + \cos 2\theta) d\theta = \frac{1}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right] + C$
 $\int \sin^2 \theta \cos^2 \theta d\theta = \int \sin \theta \cos \theta \cdot \sin \theta \cos \theta d\theta = \frac{1}{2} \int \sin 2\theta \cdot \frac{1}{2} \sin 2\theta d\theta = \frac{1}{4} \int \sin^2 2\theta d\theta = \frac{1}{4} \int (1 - \cos 4\theta) d\theta = \frac{1}{4} \left[\theta - \frac{1}{4} \sin 4\theta \right] + C$
 Total: $\frac{1}{8} \sin^8 \theta - \frac{1}{5} \sin^{10} \theta + \frac{1}{12} \sin^{12} \theta + C$

19. (12 pts) A spherical tank with radius 5 m is half full of a liquid that has a density of 1000 kg/m³. The tank has a 1 m spout at the top. Set up an integral to find the work required to pump all the water out of the spout. (Use 9.8 m/s² for g .)
- Note 1. Do NOT evaluate your integral.
 Note 2. Clearly indicate in the picture below where you are placing your axis and which direction is positive.
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- key:
- (a) If the center of tank is 0, and going up is positive direction: $W = 9800\pi \int_{-5}^0 (25 - y^2)(6-y) dy$
 (b) If the bottom of tank is 0, and going up is positive direction: $W = 9800\pi \int_0^5 (25 - (y-5)^2)(11-y) dy$
 (c) If the top of tank is 0, and going down is positive direction: $W = 9800\pi \int_0^5 (25 - (y-5)^2)(y+1) dy$
 (d) If the top of tank is 0, and going down is negative direction: $W = 9800\pi \int_{-5}^0 (25 - (y+5)^2)(1-y) dy$