MATH 152, Spring 2021

COMMON EXAM I - VERSION A

LAST NAME(print): Solutions FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

SECTION NUMBER: __________

DIRECTIONS:

1. The use of a calculator, notes, or non-approved webpages is prohibited.

2. For multiple choice questions, the answer choices are in no particular order. You will need to select the corresponding answer choice in eCampus/Canvass.

3. For workout questions, work these problems on the answer template that was provided by your instructor. If your instructor did not provide the template, use your own paper. Don't forget to write your Name and UIN at the top of the page for every work out question. Ignore the yes/no choice at the end.

4. When you are done with the exam, you will use your phone to scan the solutions to the workout questions into a single pdf file and submit this pdf file to Gradescope. **Only submit the solutions to the workout questions. Do not include any solutions to the multiple choice.**

5. Show all your work neatly and concisely. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

6. You are expected to follow THE AGGIE CODE OF HONOR **"An Aggie does not lie, cheat or steal, or tolerate those who do."**

PART I: Multiple Choice. 4 points each.

1. $\displaystyle\int \frac{\sin x}{(1+\cos x)^3}\,dx =$

   (a) $\dfrac{1}{2(1+\cos x)^2} + C$

   (b) $\dfrac{1}{(1+\cos x)^2} + C$

   (c) $\dfrac{-1}{2(1+\cos x)^2} + C$

   (d) $\dfrac{1}{4(1+\cos x)^4} + C$

   (e) $\dfrac{-1}{4(1+\cos x)^4} + C$

u-sub

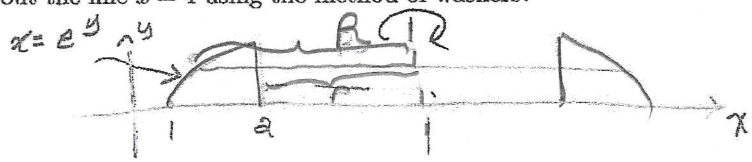
$u = 1+\cos x$

$du = -\sin x\,dx$

$-\displaystyle\int \frac{du}{u^3} = \frac{1}{2u^2} + C$

$= \boxed{\dfrac{1}{2(1+\cos x)^2} + C}$

2. Consider the region R bounded by $y = \ln x$, $y = 0$, $x = 2$. Which of the following integrals gives the volume of the solid obtained by rotating R about the line $x = 4$ using the method of washers?



   (a) $\displaystyle\int_0^{\ln(2)} \pi\left((4-e^y)^2 - 4\right)\,dy$

   (b) $\displaystyle\int_0^{2} \pi\left(4-(4-e^y)^2\right)\,dy$

   (c) $\displaystyle\int_0^{\ln(2)} \pi\left(4-(e^y-4)^2\right)\,dy$

   (d) $\displaystyle\int_0^{\ln(2)} \pi\left(16-(4-e^y)^2\right)\,dy$

   (e) $\displaystyle\int_0^{2} \pi\left((4-e^y)^2 - 16\right)\,dy$

$x=4$

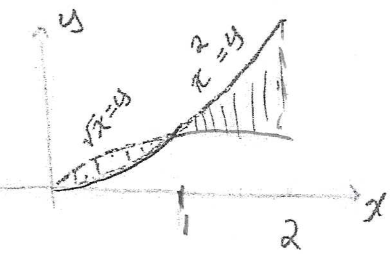
$x=2,\ y=\ln(2)$

$R = 4-e^y$

$r = 2$

$V = \displaystyle\int_0^{\ln 2} \pi\left((4-e^y)^2 - 4\right)\,dy$

3. Consider the region R bounded by $y = x^2$, $y = \sqrt{x}$ on the interval from $x = 0$ to $x = 2$. Which of the following gives the area of R?



   (a) $\displaystyle\int_0^1 (\sqrt{x}-x^2)\,dx + \int_1^2 (x^2-\sqrt{x})\,dx$

   (b) $\displaystyle\int_0^1 (x^2-\sqrt{x})\,dx + \int_1^2 (\sqrt{x}-x^2)\,dx$

   (c) $\displaystyle\int_0^2 (\sqrt{x}-x^2)\,dx$

   (d) $\displaystyle\int_0^2 (x^2-\sqrt{x})\,dx$

   (e) None of these

$A = \displaystyle\int_0^1 \left(\sqrt{x}-x^2\right)dx + \int_1^2 \left(x^2-\sqrt{x}\right)dx$

4. $\int_{-3}^{1} x\sqrt{x+3}\,dx =$

(a) $\frac{-16}{5}$

(b) $\frac{32}{5}$

(c) $\frac{112}{15}$

(d) $\frac{-56}{5}$

(e) $\frac{-116}{5}$

$u = x+3 <$ $x=1, u=4$; $x=-3, u=0$

$du = dx$

$x = u-3$

$$\int_0^4 (u-3)\sqrt{u}\,du = \int_0^4 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - 3\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_0^4$$

$$\frac{2}{5}\left(4^{5/2}\right) - 2\left(4^{3/2}\right) = \frac{2}{5}(32) - 2(8)$$

$$= \boxed{-\frac{16}{5}}$$

5. Consider the region R bounded by $y = e^{2x}$, $y = 1$ and $x = 2$. Find the area of R.

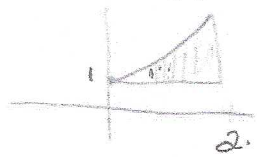
(a) $\frac{1}{2}e^4 - \frac{5}{2}$

(b) $\frac{1}{2}e^4 - \frac{3}{2}$

(c) $2e^4 - 2$

(d) $\frac{1}{2}e^4 - 2$

(e) $2e^4 - 1$



$$A = \int_0^2 \left(e^{2x} - 1\right) dx$$

$$= \left(\frac{1}{2}e^{2x} - x\right)\Big|_0^2$$

$$= \frac{1}{2}e^4 - 2 - \left(\frac{1}{2}\right)$$

$$\boxed{\frac{1}{2}e^4 - \frac{5}{2}}$$

6. A 10 foot rope weighing 50 pounds hangs vertically from a tall building. There is a 15 pound weight attached to the end of the rope. How much work is done in lifting the rope and the weight to the top of the building?

(a) 400 foot pounds

(b) 7650 foot pounds

(c) 235 foot pounds

(d) 250 foot pounds

(e) None of these

$$W = \int_0^{10} (65 - 5x)\,dx$$

$$= 65x - \frac{5}{2}x^2 \Big|_0^{10} = 650 - \frac{5}{2}(100)$$

$= 400$ Ft-lbs

7. Consider the region R bounded by $x = \sqrt{y}$ and $x = \frac{y}{2}$. Which of the following is the correct integral to find the volume obtained by rotating the region R about the line $y = -1$ using the method of shells?

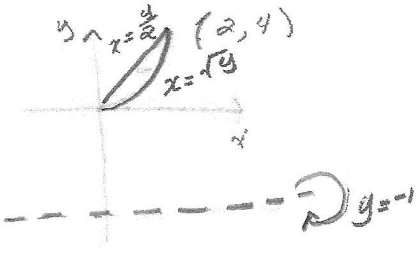
(a) $\int_0^4 2\pi(y+1)\left(\sqrt{y} - \frac{y}{2}\right) dy$

(b) $\int_0^4 2\pi(y-1)\left(\sqrt{y} - \frac{y}{2}\right) dy$

(c) $\int_0^4 2\pi(y+1)\left(\frac{y}{2} - \sqrt{y}\right) dy$

(d) $\int_0^2 2\pi(y+1)\left(\frac{y}{2} - \sqrt{y}\right) dy$

(e) $\int_0^2 2\pi(y+1)\left(\sqrt{y} - \frac{y}{2}\right) dy$



$\sqrt{y} = \frac{y}{2}$

$y = \frac{y^2}{4}$

$4y - y^2 = 0$

$y = 0;\ y = 4$

$r = y+1$

$h = \sqrt{y} - \frac{y}{2}$

$$V = \int_0^4 2\pi(y+1)\left(\sqrt{y} - \frac{y}{2}\right) dy$$

parts $u=\ln(4x)$, $dv = x\,dx$

$du = \frac{1}{x}\,dx$, $v = \frac{x^2}{2}$

8. $\int x\ln(4x)\,dx =$

(a) $\frac{x^2\ln(4x)}{2} - \frac{x^2}{4} + C$

(b) $\frac{x^2\ln(4x)}{2} - \frac{x^2}{16} + C$

(c) $x\ln(4x) - \frac{1}{4}x + C$

(d) $\frac{x^2\ln(4x)}{2} - \frac{x^2}{2} + C$

(e) $x\ln(4x) - \frac{1}{2}x^2 + C$

$\int x\ln(4x)\,dx = uv - \int v\,du$

$= \frac{x^2}{2}\ln(4x) - \int \frac{x^2}{2}\left(\frac{1}{x}\right)dx$

$= \frac{x^2}{2}\ln(4x) - \frac{1}{2}\int x\,dx$

$= x^2/2\,\ln(4x) - \frac{1}{4}x^2 + C$

9. A force of 30 N is required to stretch a spring from its natural length of 1 m to a length of 3 m. How much work is done in stretching the spring from 2 m to 4 m?

(a) 60 Joules

(b) $\frac{221}{2}$ Joules

(c) 240 Joules

(d) 360 Joules

(e) 90 Joules

Given $f(2)=30$, where $f(x)$ = force function

$2k=30$, so $k=15$

$f(x) = 15x$, so $W = \int_1^3 15x\,dx$

$= 15\frac{x^2}{2}\Big|_1^3 = 15\left(\frac{9}{2}-\frac{1}{2}\right) = \boxed{60\text{ J}}$

10. Consider the region R bounded by $y=\sqrt{x}$, $y=1$, $x=0$. Find the volume obtained by rotating the region R about the line $y=1$.

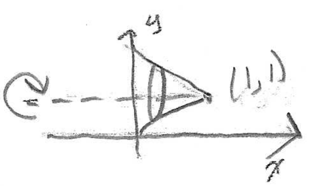
(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{2}$

(c) $\frac{7\pi}{6}$

(d) $\frac{\pi}{3}$

(e) $\frac{5\pi}{6}$



$V = \int_0^1 \pi(1-\sqrt{x})^2\,dx$

$= \pi\int_0^1 (1-2\sqrt{x}+x)\,dx$

$= \pi\left(x - 2\cdot\frac{2}{3}x^{3/2} + \frac{x^2}{2}\right)\Big|_0^1$

$= \pi\left(1-\frac{4}{3}+\frac{1}{2}\right)$

$= \boxed{\frac{\pi}{6}}$

11. $\int_0^{\pi/2} \cos^3 x\sin^3 x\,dx =$

(a) $\frac{1}{12}$

(b) $\frac{2}{15}$

(c) $\frac{5}{12}$

(d) $\frac{-1}{12}$

(e) $\frac{-2}{15}$

$\int_0^{\frac{\pi}{2}} \cos^2 x\sin^3 x\cos x\,dx$

$\int_0^{\frac{\pi}{2}} (1-\sin^2 x)\sin^3 x\cos x\,dx$ $u=\sin x$, $du = \cos x\,dx$

$\int_0^1 (1-u^2)u^3\,du = \int_0^1 (u^3-u^5)\,du$

$= \frac{1}{4}-\frac{1}{6}$ or $\boxed{\frac{1}{12}}$

12. $\int \tan^4 x \sec^4 x \, dx =$

(a) $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$

(b) $\frac{\tan^7 x}{7} - \frac{\tan^5 x}{5} + C$

(c) $\frac{\tan^9 x}{9} + \frac{\tan^5 x}{5} + C$

(d) $\frac{\tan^9 x}{9} - \frac{\tan^5 x}{5} + C$

(e) None of these

$\int \tan^4 x \sec^2 x \sec^2 x \, dx$

$\int \tan^4 x (\tan^2 x + 1) \sec^2 x \, dx$

$u = \tan x \rightarrow du = \sec^2 x \, dx$

$\int u^4 (u^2 + 1) \, du = \int (u^6 + u^4) \, du$

$= \frac{u^7}{7} + \frac{u^5}{5} + C$

$= \boxed{\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C}$

13. $\int_0^1 (x+2) \cos x \, dx =$

(a) $3\sin(1) + \cos(1) - 1$

(b) $3\sin(1) + \cos(1)$

(c) $3\sin(1) - \cos(1) - 1$

(d) $3\sin(1) - \cos(1)$

(e) None of these

$u = x+2 \quad dv = \cos x \, dx$

$du = dx \quad v = \sin x$

$(x+2)\sin x \Big|_0^1 - \int_0^1 \sin x \, dx$

$3\sin(1) + \cos x \Big|_0^1$

$\boxed{3\sin(1) + \cos(1) - 1}$

14. $\int_0^{\pi/3} \tan(x) \, dx =$

(a) $-\ln\left(\frac{1}{2}\right)$

(b) $\ln\left(\frac{1}{2}\right)$

(c) $-\ln\left(\frac{\sqrt{3}}{2}\right)$

(d) $\ln(\sqrt{2})$

(e) $\ln(3)$

$\int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x} \, dx$

$u = \cos x$ $\quad x = \frac{\pi}{3}, u = \frac{1}{2}$ $\quad x = 0, u = 1$

$du = -\sin x \, dx$

$-\int_1^{\frac{1}{2}} \frac{du}{u} = \int_{\frac{1}{2}}^1 \frac{du}{u} = \ln|u| \Big|_{\frac{1}{2}}^1 = \cancel{\ln(1)} - \ln(\tfrac{1}{2})$

$= \boxed{-\ln\left(\frac{1}{2}\right)}$

15. $\int_0^{\pi/4} \sin^2(x) \, dx =$

(a) $\frac{\pi}{8} - \frac{1}{4}$

(b) $\frac{\pi}{8}$

(c) $\frac{\pi}{8} - \frac{1}{2}$

(d) $\frac{2}{\sqrt{2}} - \frac{\pi}{2\sqrt{2}}$

(e) $\frac{\pi}{8} + \frac{1}{4}$

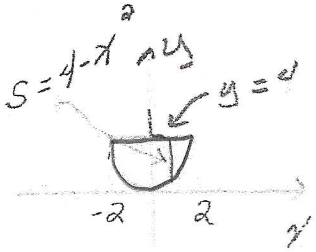
$\int_0^{\frac{\pi}{4}} \frac{1}{2}(1 - \cos 2x) \, dx$

$\frac{1}{2}(x - \frac{1}{2}\sin 2x) \Big|_0^{\frac{\pi}{4}}$

$\frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \boxed{\frac{\pi}{8} - \frac{1}{4}}$

PART II: Free Response: Show all work and box your final answer!

16. (8 pts) Consider the solid S described here. The base of S is the region bounded by $y = x^2$ and $y = 4$. Cross sections perpendicular to the x - axis are squares. Find the volume of S.



The side of an arbitrary square has a variable length of $4 - x^2$.

$$V = \int_{-2}^{2} (4 - x^2)^2 \, dx, \text{ or, using symmetry,}$$

$$V = 2\int_0^2 (4 - x^2)^2 \, dx$$

$$= 2\int_0^2 (16 - 8x^2 + x^4) \, dx$$

$$= 2\left(16x - \frac{8x^3}{3} + \frac{x^5}{5}\right)\Big|_0^2$$

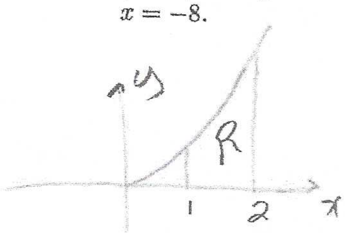
$$= 2\left(32 - \frac{64}{3} + \frac{32}{5}\right)$$

$$= 2\left(\frac{256}{15}\right) \text{ or } \boxed{\frac{512}{15}}$$

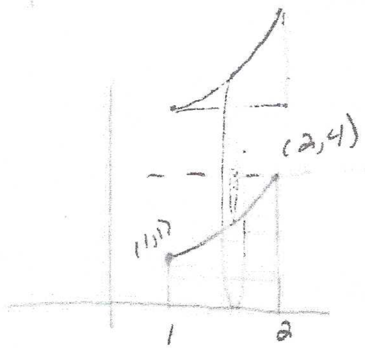
17. Consider the region R bounded by $y = x^2$, $y = 0$, $x = 1$ and $x = 2$.

(a) (8 pts) Set up but **do not evaluate** the integral that gives the volume obtained by rotating R about the line $y = 4$.

(b) (8 pts) Set up but **do not evaluate** the integral that gives the volume obtained by rotating R about the line $x = -8$.



(a)



$y=4$

• washers: $R = 4,\ r = 4 - x^2$

ideal method

$$\boxed{V = \pi\int_1^2 \left(16 - (4-x^2)^2\right) dx}$$

• shells requires two integrals:

for $0 \le y \le 1$, $r = 4-y$, $h = 1$

for $1 \le y \le 4$, $r = 4-y$, $h = 2-\sqrt{y}$

$$V = \int_0^1 2\pi(4-y)(1)\,dy + \int_1^4 2\pi(4-y)(2-\sqrt{y})\,dy$$

(b)

• shells: $r = x+8$, $h = x^2$

$$\boxed{V = \int_1^2 2\pi(x+8)x^2\,dx}$$

ideal method

$x = -8$

• washers requires two integrals:

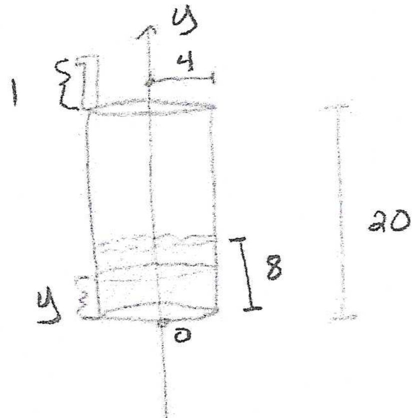
for $0 \le y \le 1$, $R = 10$, $r = 9$

for $1 \le y \le 4$, $R = 10$, $r = \sqrt{y} + 8$

$$V = \int_0^1 \pi(100 - 81)\,dy + \int_1^4 \pi\left(100 - (\sqrt{y}+8)^2\right) dy$$

$$= \int_0^1 \pi(19)\,dy + \int_1^4 \pi\left(100 - (\sqrt{y}+8)^2\right) dy$$

18. (8 pts) A cylindrical shaped tank is filled with water to a depth of 8 m. The tank has a height of 20 meters and a radius of 4 meters. Set up but **do not evaluate** an integral that gives the work required to pump all of the water out of a 1 meter tall spout top located at the of the tank. Use $\rho g = 9800 N/m^3$ for the weight density of water. **Clearly indicate where you are placing the axis and which direction is positive.**



- volume of a slice of water is $V_s = \pi(4)^2 \Delta y$
- Force acting on this slice in order for it to be pumped up is

$$F_s = 16\pi \rho g \Delta y, \text{ where } \rho g = 9800 N/m^3$$

- Distance slice moves to exit the 1m spout is $d = 20 - y + 1 = 21 - y$
- work done to move slice out a 1m spout is $W_s = F_s \cdot d$

$$= 16\pi \rho g (21 - y)\Delta y$$

$$\boxed{W = \int_0^8 16\pi \rho g (21 - y)\, dy}$$

use identity $\cos^2 x = \frac{1}{2}(1+\cos(2x))$

19. (8 pts) Find $\int \cos^4 x \, dx$

$$\int (\cos^2 x)^2 \, dx$$

$$= \int \left[\tfrac{1}{2}(1+\cos(2x)) \right]^2 dx$$

$$\tfrac{1}{4}\int \left(1 + 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \tfrac{1}{4}\int \left(1 + 2\cos(2x) + \tfrac{1}{2}(1+\cos(4x))\right) dx$$

$$= \tfrac{1}{4}\int \left(\tfrac{3}{2} + 2\cos(2x) + \tfrac{1}{2}\cos(4x)\right) dx$$

$$= \tfrac{1}{4}\left(\tfrac{3}{2}x + 2\cdot\tfrac{1}{2}\sin(2x) + \tfrac{1}{8}\sin(4x)\right) + C$$

$$= \tfrac{1}{4}\left(\tfrac{3}{2}x + \sin(2x) + \tfrac{1}{8}\sin(4x)\right) + C$$